5.5 The Quadratic Formula

Ex. 1 Find the roots/zeroes/x-int of $y = 2x^2 - 11x + 5$.

→ standard form
→ factor

$$0 = (2x-1)(x-5)$$

$\boxed{x = \frac{1}{2}}$ $\boxed{x = 5}$

M: 10
A: −11
N: −1, −10

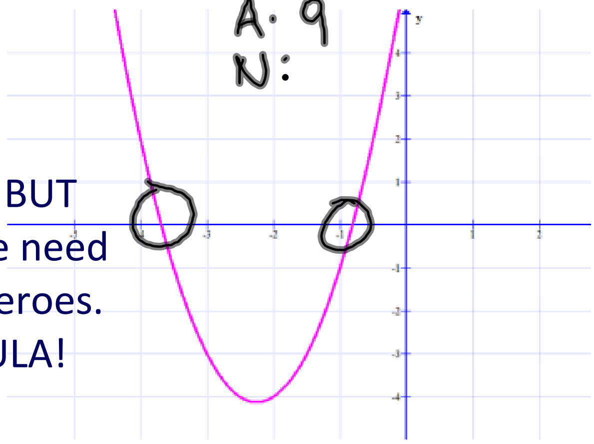
$\frac{2x^2}{-x}$ $\frac{2x^2}{-10x}$

Can you find the zeroes of $y = 2x^2 + 9x + 6$?

Prime

M: 12
A: 9
N:

The equation cannot be factored BUT you can see it has two zeroes. We need another method for finding the zeroes. We need the QUADRATIC FORMULA!



5.5 The Quadratic Formula

x - intercepts → expand

→ factored form

Ex. 2 Find the zeroes of y = 2(x - 1)^2 - 18.

→ vertex form

Solve directly

BEDMAS ←

$$0 = 2(x-1)^2 - 18$$

$$\frac{18}{2} = \frac{\cancel{2}(x-1)^2}{\cancel{2}}$$

$$9 = (x-1)^2$$

$$\pm\sqrt{9} = \sqrt{(x-1)^2}$$

$$\pm 3 = (x-1)$$

$$1 \pm 3 = x$$

$x = 1+3$

$\boxed{x = 4}$

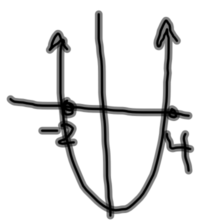
$x = 1-3$

$\boxed{x = -2}$

$2(x-1)^2$

↓

$x = 1$



Solve for x by completing the square $2x^2 - 11x + 5 = 0$.

$$\left(-\frac{11}{2}\div 2\right)^2 = \left(-\frac{11}{2}\times\frac{1}{2}\right)^2 = \left(-\frac{11}{4}\right)^2 = \frac{121}{16}$$

$$2\left(x^2 - \frac{11}{2}x\right) + 5 = 0$$

$$2\left(x^2 - \frac{11}{2}x + \frac{121}{16} - \frac{121}{16}\right) + 5 = 0$$

$$2\left(x^2 - \frac{11}{2}x + \frac{121}{16}\right) - \frac{121}{\cancel{16}\ 8}(\cancel{2}) + 5 = 0$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + \frac{40}{8} = 0$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{81}{8} = 0$$

$$\frac{2\left(x - \frac{11}{4}\right)^2}{2} = \frac{81}{8}\times\frac{1}{2}$$

$$\left(x - \frac{11}{4}\right)^2 = \frac{81}{16}$$

$$\left(x - \frac{11}{4}\right) = \pm\sqrt{\frac{81}{16}}$$

$$x - \frac{11}{4} = \pm\frac{9}{4}$$

$$x = \frac{11}{4} \pm \frac{9}{4}$$

$$x = \frac{11}{4} + \frac{9}{4} \qquad x = \frac{11}{4} - \frac{9}{4}$$

$$x = \frac{20}{4} \qquad x = \frac{2}{4}$$

$$\boxed{x = 5} \qquad \boxed{x = \frac{1}{2}}$$

To derive the Quadratic Formula solve for x if $ax^2 + bx + c = 0$ by completing the square!

$$\left(\frac{b}{a}\div 2\right)^2 \quad \left(\frac{b}{a}\times\frac{1}{2}\right)^2 \quad \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^{\cancel{2}}}(\cancel{a}) + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4ac}{4a} = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right) = 0$$

Solve for 'x'

$$\frac{\cancel{a}\left(x + \frac{b}{2a}\right)^2}{\cancel{a}} = \frac{\frac{b^2 - 4ac}{4a}}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2 - 4ac}{4a}\right)\times\frac{1}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

X-intercepts

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

To find the roots when the equation is in standard form and DOES NOT factor use:

The Quadratic Formula:

For $ax^2 + bx + c = 0$,

roots : $$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$b^2 - 4ac$ is called the DISCRIMINANT

$b^2 - 4ac$

Ex. 3 Solve. Give EXACT solutions then decimal approximations.

a) $0 = x^2 - 3x + 1$

$a=1 \quad b=-3 \quad c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

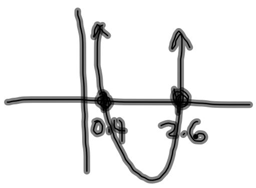
$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$$\boxed{x = \frac{3+\sqrt{5}}{2}} \qquad \boxed{x = \frac{3-\sqrt{5}}{2}}$$

Exact Solutions

$$\boxed{x \doteq 2.6} \qquad \boxed{x \doteq 0.38}$$



b) $2x(x - 3) = 7$ $\rightarrow \neq 0$

~~$x=0$ $\quad$ $x=3$~~

$2x(x-3) = 7$

$2x^2 - 6x - 7 = 0$

$a=2 \quad b=-6 \quad c=-7$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2-4(2)(-7)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{36+56}}{4}$$

$$= \frac{6 \pm \sqrt{92}}{4}$$

$$\boxed{x = \frac{6+\sqrt{92}}{4}} \qquad \boxed{x = \frac{6-\sqrt{92}}{4}}$$

$x \doteq 3.9 \qquad x = -0.9$

gr 11

$\sqrt{92}$

$= \sqrt{4 \times 23}$

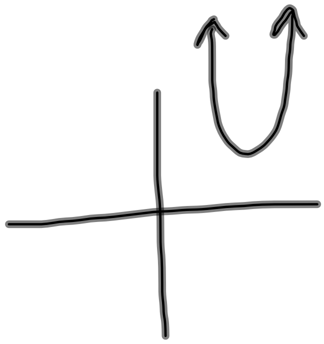
$= 2\sqrt{23}$

Ex. 4 Solve each of the following using the quadratic formula:

$3x^2+2x + 15 = 0$ $a = 3$ $b = 2$ $c = 15$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm\sqrt{2^2-4(3)(15)}}{2(3)}$$



$$=\frac{-2\pm\sqrt{4-180}}{6}$$

$$=\frac{-2\pm\sqrt{-176}}{6}$$ $\longrightarrow$ can't take $\sqrt{\ }$ of neg #

NO REAL ROOT

Is there an easier way to determine the number of zeroes?

http://www.youtube.com/watch?v=O8ezDEk3qCg

Homework

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2 (check with desmos), 4, 9def