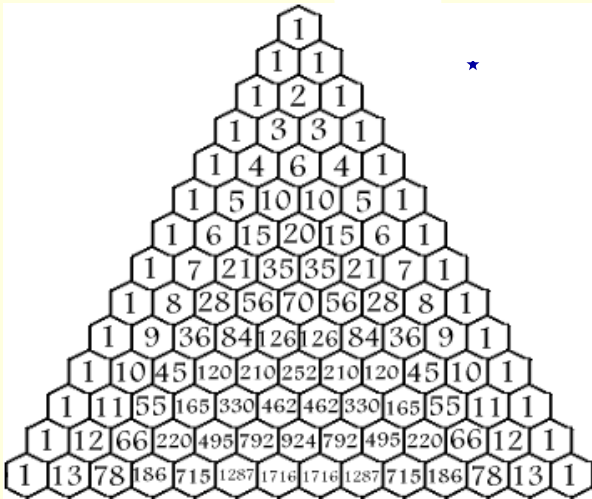


## 6.7 Pascal's Triangle

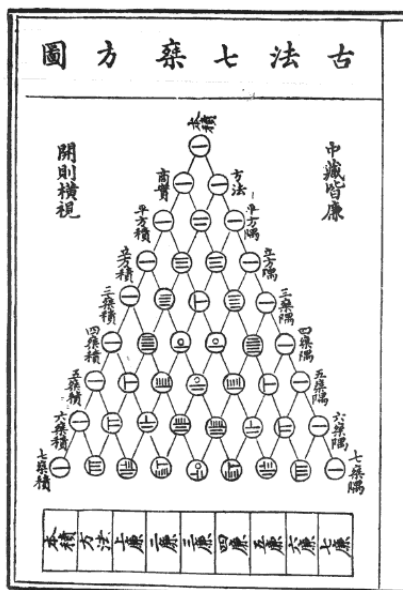


<https://www.youtube.com/watch?v=Y>

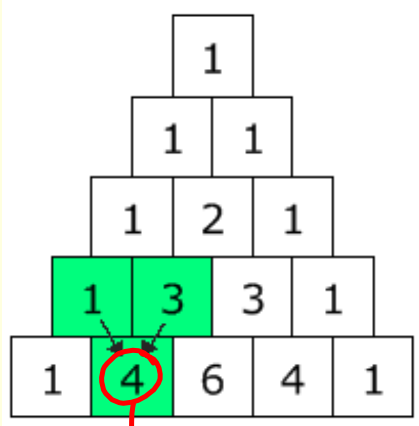
Pascal's Triangle is named after mathematician, Blaise Pascal (1623-1662).

He discovered the triangle and many of its applications at age 13. Since he published his findings, western history attaches his name to the triangle.

Pascal was not the first to know this triangle though. Chinese mathematician, Chu Shi-kie also wrote of the triangle and many of its applications in the 14<sup>th</sup> century.



Chu Shi-kie's triangle, 700 years old,  
300 years older than Pascal!



Row 4  
Position 1

### In Pascal's Triangle:

- Each term is equal to the sum of the two terms immediately above it.
- The term at the end of each row is 1.
- Rows and positions in rows are numbered starting from ZERO.  
∴ the 3<sup>rd</sup> row is actually Row 2.
- Also, the term equal to 4 is found in row 4, position 1.

$t_{r,n}$  ← row position  
 $t_{4,1} = 4$

As terms we have:

$$\begin{array}{cccc}
 & & t_{0,0} & & \\
 & & t_{1,0} & t_{1,1} & \\
 & & t_{2,0} & + & t_{2,1} & t_{2,2} \\
 & t_{3,0} & t_{3,1} & & t_{3,2} & t_{3,3} \\
 & & \text{etc...} & & & \\
 t_{4,0} & t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4}
 \end{array}$$

$t_{n,r}$  represents the term in row  $n$ , position  $r$ .

Thus,  $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$

$$t_{15,3} = t_{14,2} + t_{14,3}$$

Pascal's Triangle isn't just cool for the sake of being cool...it has the most amazing properties and applications. For instance:

### ROW SUMS

The sum of the terms in any row  $n$  in Pascal's Triangle is  $2^n$ .

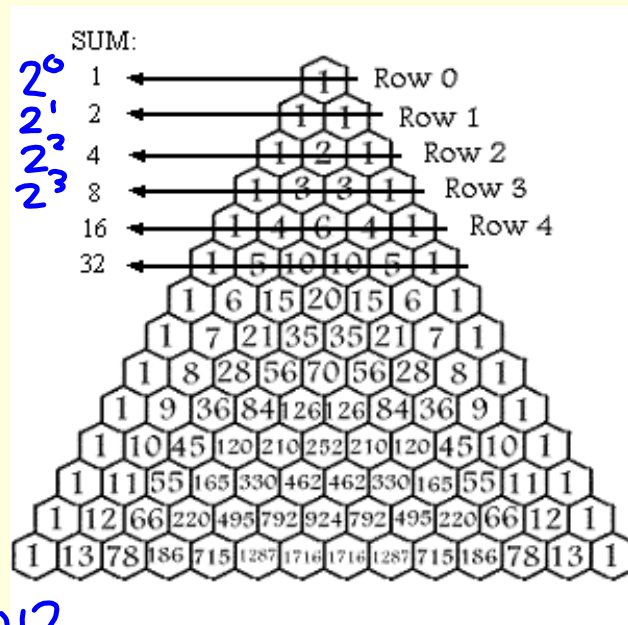
eg. sum of row 9 =  $2^9 = 512$

eg. row whose sum is 4096

$\therefore 2^n = 4096$

$2^n = 4096$

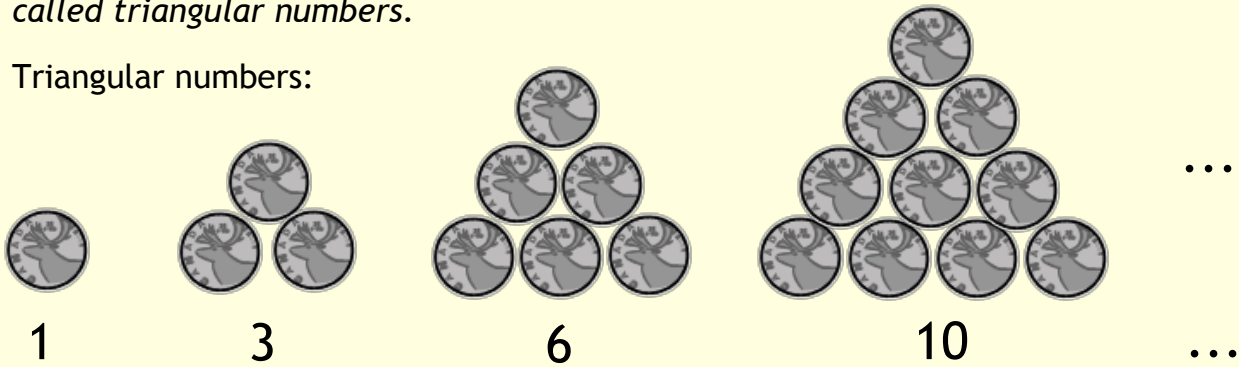
$2^{12} = 4096 \therefore \text{Row } 12$



# TRIANGULAR NUMBERS

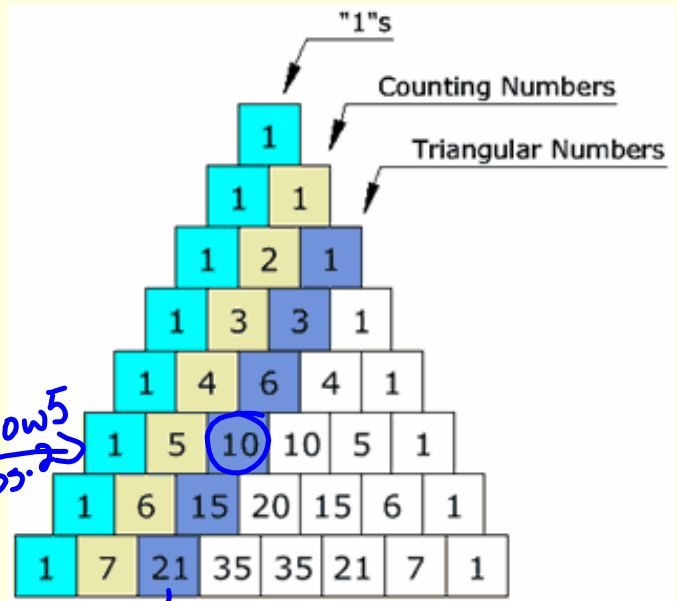
Numbers that correspond to the number of items stacked in a triangular array are called triangular numbers.

Triangular numbers:

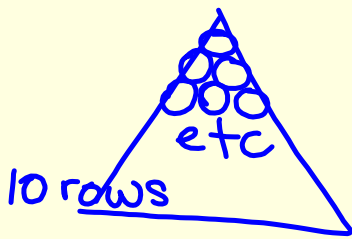


We find the list of triangular numbers in the third diagonal of Pascal's Triangle.

Ex 2. Relate Pascal's Triangle to the number of coins in a triangle with  $n$  rows.



$\Delta \# 4$   
 $n=4$   
 $t_{n+1,2}$   
 $t_{5,2}$



Row 5  
 Pos. 2  
 $t_{6+1,2}$   
 $7C2 = 21$

$t_{10+1,2}$   
 $t_{11,2}$   
 $nCr$   
 $11C2 = 55$

## FIBONACCI NUMBERS

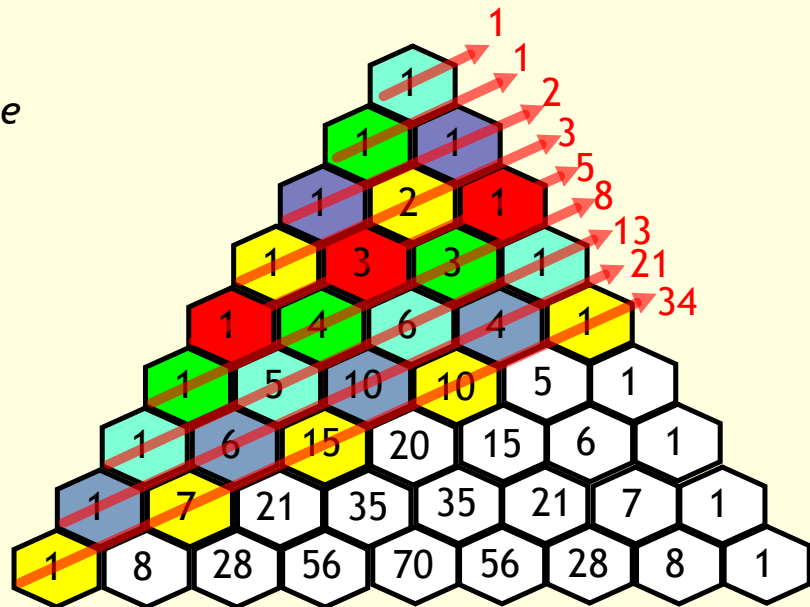
---

*The fibonacci sequence begins with 1,1, and then each subsequent term is found by taking the sum of the previous two terms.*

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377,...

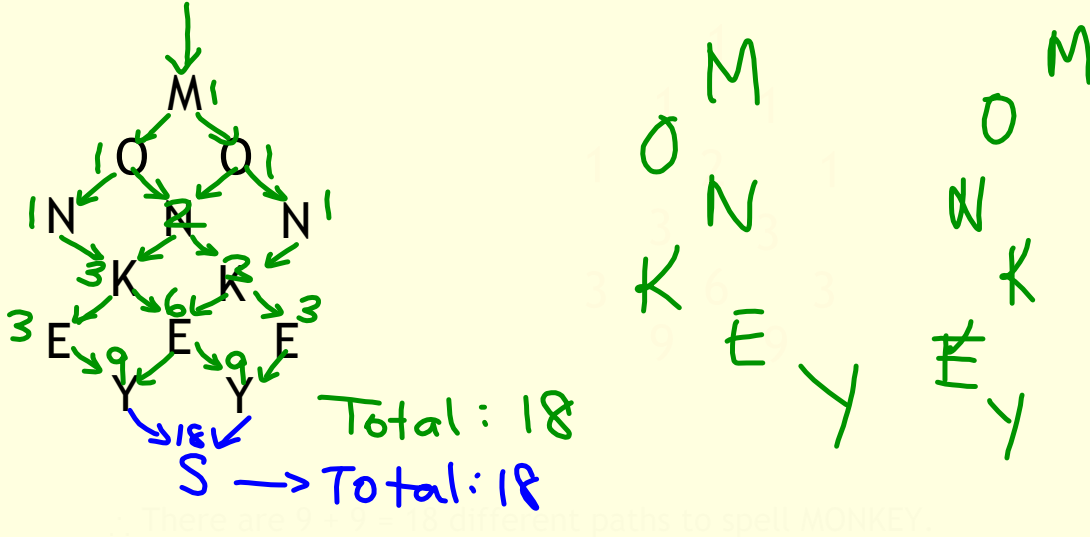
*The fibonacci numbers are found by adding the terms of the diagonals of Pascal's Triangle:*

*Fibonacci numbers are found in the strangest places: the golden ratio, nature, The DaVinci Code,...*



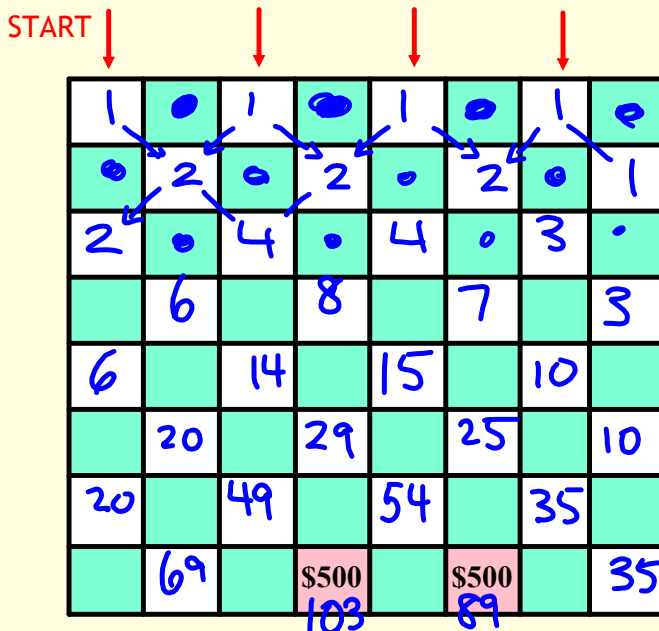
Pascal's Triangle is often used to determine the number of paths between points.

Example 1: How many paths will spell MONKEY by starting at the top and proceeding downwards, moving diagonally left or right?



Example 2:

In Plinko, a token slides down a board. If the token cannot go through a shaded square, in how many ways could you win \$500?



Total  
= 69 + 103 +  
89 + 35  
= 296

$P(\text{win } \$500)$   
=  $\frac{192}{296}$   
= 65%

192 ways  
of winning \$500

6.7 - More on Pascal's Triangle (lead up to Binomial Theorem)

COPY ON SEPERATE PAPER

There is a more efficient way of finding a specific number within the triangle without drawing it out. We can use **Combinatorics** (a type of Math used mostly for graph theory and probability theory) to help us.

Definition of a Combination

$${}^n C_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

factorial (the product of all natural numbers less than or equal to n)

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

pronounced "n choose r"

On your calculators, nCr button can be used instead of using the formal algebraic definition

In the case of Pascal's Triangle, **n CHOOSE r** means find the number that is in row n, position r in the triangle.

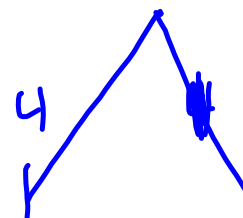
Ex. 1 Evaluate the following combinations.

a)  ${}^9 C_7$   
 Row 9, position 7  
 $= 36$

b)  $\binom{7}{3}$   
 ← Row  
 ← Pos  
 $= {}^7 C_3$   
 $= 35$

c)  $\binom{6}{0}$   
 $= {}^6 C_0$   
 $= 1$

d)  $\binom{4}{4}$   
 $= 1$



Ex. 2

a) find the term in row 2, position 2 of Pascal's Triangle

$n=2$     $r=2$   
 ${}^2 C_2 = 1$

b) find the term in the 3rd row, 3rd position in Pascal's Triangle

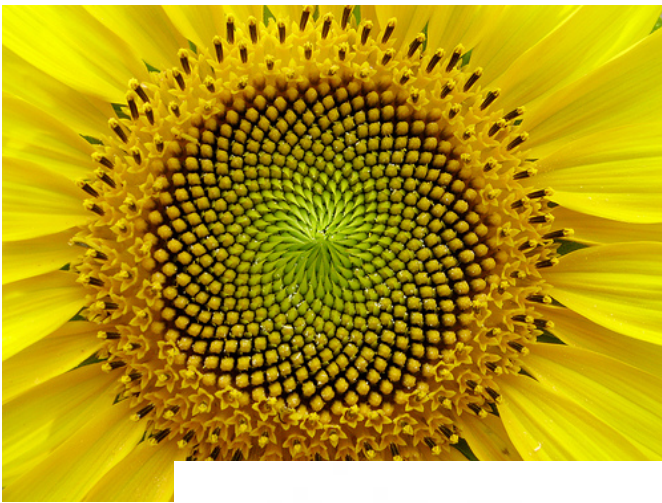
0	→	↓	1st	
1	→	1	2nd	
2	→	1	2	1
		1st	2nd	3rd

$n=2$     $r=2$   
 ${}^2 C_2 = 1$





# Homework: Handout



#5

