

1.4B: Working with Radicals - Day 2

Ex. 1 Multiply each of the following:

$$\text{a) } 4\sqrt{5}(2\sqrt{8}-3\sqrt{5})$$

$$= 8\sqrt{40} - 12(5)$$

$$= 8\sqrt{4 \cdot 10} - 60$$

$$= 8 \cdot 2\sqrt{10} - 60$$

How? Distributive Property.
May need to simplify after multiplying.

$$\rightarrow 16\sqrt{10} - 60$$

$$\text{b) } (2\sqrt{3}-\sqrt{5})(4\sqrt{3}+2\sqrt{5})$$

$$= 8(3) + \underline{4\sqrt{15} - 4\sqrt{15}} - 2(5)$$

$$= 14$$

$$\text{c) } (2\sqrt{5}-\sqrt{3})^2$$

$$= (2\sqrt{5}-\sqrt{3})(2\sqrt{5}-\sqrt{3})$$

$$= 4(5) - \underline{2\sqrt{15} - 2\sqrt{15}} + 3$$

$$= 23 - 4\sqrt{15}$$

Ex. 2 Simplify each of the following:

a) $\frac{12+3\sqrt{12}}{4}$



$\frac{12+3\sqrt{12}}{4}$
 $\frac{12+6\sqrt{3}}{4}$

$\frac{4}{4}$

$\frac{6+3\sqrt{3}}{2}$ ★

$3 + \frac{3\sqrt{3}}{2}$

$\frac{12+6\sqrt{3}}{4} = \frac{2(6+3\sqrt{3})}{2}$

★ What is the GCF between 4, 6, 12?

b) $\frac{15 \pm \sqrt{27}}{3}$

← ● Look familiar?

$\frac{15 \pm 3\sqrt{3}}{3}$

$\frac{3}{3}$

$\frac{3(5 \pm \sqrt{3})}{3}$

$\frac{3}{3}$

$5 \pm \sqrt{3}$

Ex. 3 Simplify - Rationalizing Denominators

$$\begin{aligned} \text{a) } & \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ & = \frac{2\sqrt{5}}{5} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{3\sqrt{5}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & = \frac{3\sqrt{10}}{4(2)} \\ & = \frac{3\sqrt{10}}{8} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{5\sqrt{10}}{15\sqrt{20}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & = \frac{1\sqrt{10}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & = \frac{\sqrt{10}}{20} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & = \frac{\sqrt{10}}{\sqrt{20}} \\ & = \frac{\sqrt{10}}{\sqrt{10 \cdot 2}} \\ & = \frac{\sqrt{10}}{\sqrt{10} \sqrt{2}} \\ & = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & = \frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{1}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \\ & = \frac{(\sqrt[3]{2})^2}{2} \end{aligned}$$

Simplify

$$\begin{aligned} \text{e) } & \frac{1}{\sqrt[3]{32}} \leftarrow \begin{aligned} & \beta = 1 \\ & 2^3 = 8 \\ & 3^3 = 27 \\ & 4^3 = 64 \end{aligned} \\ & = \frac{1}{\sqrt[3]{8 \cdot 4}} \\ & = \frac{1}{2\sqrt[3]{4}} \times \frac{\sqrt[3]{4}}{\sqrt[3]{4}} \times \frac{\sqrt[3]{4}}{\sqrt[3]{4}} \\ & = \frac{(\sqrt[3]{4})^2}{2(4)} \\ & = \frac{(\sqrt[3]{4})^2}{8} \end{aligned}$$

What if the denominator is a binomial?

e) $\frac{5}{(2\sqrt{6}-\sqrt{3})} \times \frac{2\sqrt{6}+\sqrt{3}}{(2\sqrt{6}+\sqrt{3})}$ **You must multiply by the conjugate.**
 The conjugate of $a + b$ is $a - b$.
 Change the sign between the two terms.

$$= \frac{10\sqrt{6} + 5\sqrt{3}}{4(6) + \cancel{2\sqrt{18}} - \cancel{2\sqrt{18}} - 3}$$

$$= \frac{10\sqrt{6} + 5\sqrt{3}}{21}$$

Why conjugates?
 See a familiar pattern?

f) $\frac{(\sqrt{2}+\sqrt{5})}{\sqrt{6}-\sqrt{10}} \times \frac{(\sqrt{6}+\sqrt{10})}{\sqrt{6}+\sqrt{10}}$

$$= \frac{\sqrt{12} + \sqrt{20} + \sqrt{30} + \sqrt{50}}{6 - 10}$$

$$= \frac{2\sqrt{3} + 2\sqrt{5} + \sqrt{30} + 5\sqrt{2}}{6 - 10}$$

$$= -\frac{2\sqrt{3} + 2\sqrt{5} + \sqrt{30} + 5\sqrt{2}}{4}$$

OR

$$= \frac{-2\sqrt{3} - 2\sqrt{5} - \sqrt{30} - 5\sqrt{2}}{4}$$

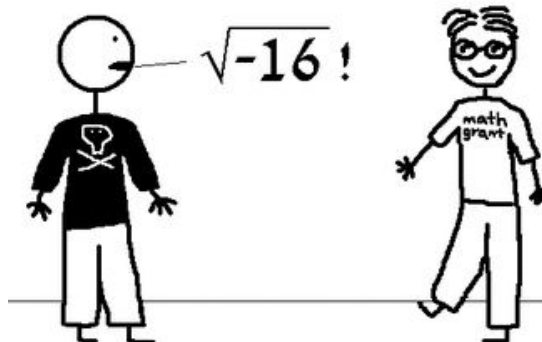
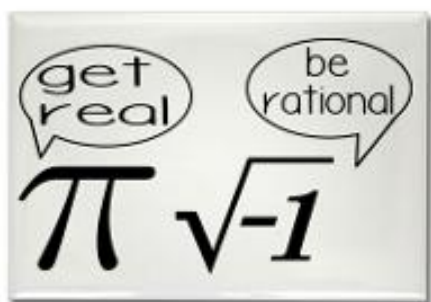
Homework

p. 39 #7cdef, 8bc, 12, 15, 16, 17, 20

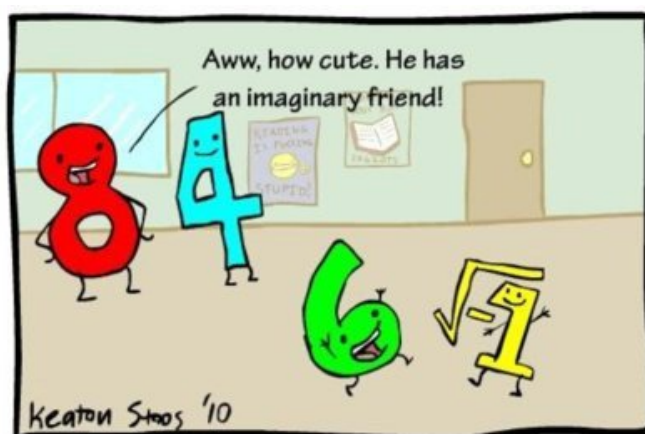
+ Handout → you need to be able to do it all



#4, 5, 13, 14



Mathematical Insults



1.4B Working with Radicals Handout

1. a) $\sqrt{12} + \sqrt{27}$
 b) $\sqrt{20} + \sqrt{45}$
 c) $\sqrt{18} - \sqrt{8}$
 d) $\sqrt{50} + \sqrt{98} - \sqrt{2}$
 e) $\sqrt{75} + \sqrt{48} + \sqrt{27}$
 f) $\sqrt{54} + \sqrt{24} + \sqrt{72} - \sqrt{32}$
 g) $\sqrt{28} - \sqrt{27} + \sqrt{63} + \sqrt{300}$

2. a) $\sqrt{2}(\sqrt{10} + 4)$
 b) $\sqrt{3}(\sqrt{6} - 1)$
 c) $\sqrt{6}(\sqrt{2} + \sqrt{6})$
 d) $2\sqrt{2}(3\sqrt{6} - \sqrt{3})$
 e) $\sqrt{2}(\sqrt{3} + 4)$
 f) $3\sqrt{2}(2\sqrt{6} + \sqrt{10})$
 g) $(\sqrt{5} + \sqrt{6})(\sqrt{5} + 3\sqrt{6})$
 h) $(2\sqrt{3} - 1)(3\sqrt{3} + 2)$
 i) $(4\sqrt{7} - 3\sqrt{2})(2\sqrt{7} + 5\sqrt{2})$
 j) $(3\sqrt{3} + 1)^2$
 k) $(2\sqrt{2} - \sqrt{5})^2$
 l) $(2 + \sqrt{3})(2 - \sqrt{3})$
 m) $(\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})$
 n) $(2\sqrt{7} + 3\sqrt{5})(2\sqrt{7} - 3\sqrt{5})$

3. a) $\frac{1}{\sqrt{3}}$
 d) $\frac{\sqrt{1}}{\sqrt{2}}$
 g) $\frac{4\sqrt{2}}{\sqrt{8}}$
 j) $\frac{3\sqrt{6}}{4\sqrt{10}}$

4. a) $\frac{1}{\sqrt{2} + 2}$
 c) $\frac{\sqrt{2}}{\sqrt{6} - 3}$
 e) $\frac{3}{\sqrt{5} - \sqrt{2}}$
 g) $\frac{2\sqrt{6}}{2\sqrt{6} + 1}$
 i) $\frac{\sqrt{2} + \sqrt{5}}{\sqrt{6} - \sqrt{10}}$

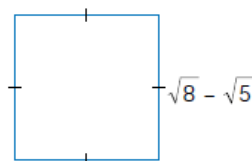
5. a) $\sqrt[3]{16} + \sqrt[3]{54}$
 c) $2(\sqrt[3]{32}) + 5(\sqrt[3]{108})$
 e) $\sqrt[3]{16} - \sqrt[3]{54}$
 g) $2(\sqrt[3]{40}) - \sqrt[3]{5}$

- b) $\frac{2}{\sqrt{5}}$
 e) $\frac{5\sqrt{5}}{2\sqrt{3}}$
 h) $\frac{3\sqrt{5}}{\sqrt{3}}$
 k) $\frac{7\sqrt{11}}{2\sqrt{3}}$

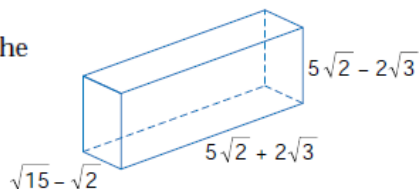
- b) $\frac{3}{\sqrt{5} - 1}$
 d) $\frac{2}{\sqrt{6} + \sqrt{3}}$
 f) $\frac{\sqrt{3}}{\sqrt{3} + \sqrt{2}}$
 h) $\frac{\sqrt{2} - 1}{\sqrt{2} + 1}$
 j) $\frac{2\sqrt{7} + \sqrt{5}}{3\sqrt{7} - 2\sqrt{5}}$

- b) $\sqrt[3]{24} + \sqrt[3]{81}$
 d) $\sqrt[3]{54} + 5(\sqrt[3]{16})$
 f) $\sqrt[3]{108} - \sqrt[3]{32}$
 h) $5(\sqrt[3]{48}) - 2(\sqrt[3]{162})$

13. **Measurement** Write and simplify an expression for the area of the square.



14. **Measurement** Express the volume of the rectangular prism in simplest radical form.



Answers

1. a) $5\sqrt{3}$ b) $5\sqrt{5}$ c) $\sqrt{2}$ d) $11\sqrt{2}$ e) $12\sqrt{3}$ f) $5\sqrt{6} + 2\sqrt{2}$ g) $5\sqrt{7} + 7\sqrt{3}$
 2. a) $2\sqrt{5} + 4\sqrt{2}$ b) $3\sqrt{2} - \sqrt{3}$ c) $2\sqrt{3} + 6$ d) $12\sqrt{3} - 2\sqrt{6}$ e) $\sqrt{6} + 4\sqrt{2}$ f) $12\sqrt{3} + 6\sqrt{5}$ g) $23 + 4\sqrt{30}$ h) $16 + \sqrt{3}$ i) $26 + 14\sqrt{14}$ j) $28 + 6\sqrt{3}$ k) $13 - 4\sqrt{10}$ l) 1 m) 4 n) -17
 3. a) $\frac{\sqrt{3}}{3}$ b) $\frac{2\sqrt{5}}{5}$ c) $\frac{2\sqrt{7}}{7}$ d) $\frac{\sqrt{2}}{2}$ e) $\frac{5\sqrt{15}}{6}$ f) $\frac{2}{3}$ g) $2\sqrt{2}$ h) $\sqrt{15}$ i) $\sqrt{2}$ j) $\frac{3\sqrt{15}}{20}$ k) $\frac{7\sqrt{33}}{6}$ l) $\frac{\sqrt{10}}{5}$
 4. a) $\frac{2 - \sqrt{2}}{2}$ b) $\frac{3 + 3\sqrt{5}}{4}$ c) $\frac{3\sqrt{2} + 2\sqrt{3}}{3}$ d) $\frac{2\sqrt{6} - 2\sqrt{3}}{3}$ e) $\sqrt{5} + \sqrt{2}$ f) $3 - \sqrt{6}$ g) $\frac{24 - 2\sqrt{6}}{23}$ h) $3 - 2\sqrt{2}$ i) $\frac{5\sqrt{2} + 2\sqrt{3} + 2\sqrt{5} + \sqrt{30}}{4}$ j) $\frac{52 + 7\sqrt{35}}{43}$
 5. a) $5\sqrt[3]{2}$ b) $5\sqrt[3]{3}$ c) $19\sqrt[3]{4}$ d) $13\sqrt[3]{2}$ e) $-\sqrt[3]{2}$ f) $\sqrt[3]{4}$ g) $3\sqrt[3]{5}$ h) $4\sqrt[3]{6}$ i) 13 j) $13 - 4\sqrt{10}$ k) $38\sqrt{15} - 38\sqrt{2}$