2.1 Midpoint and Review of y = mx + b

Remember…

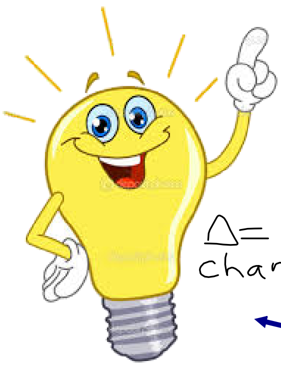
$$y = mx + b$$

To write the equation of a line you need slope and y-intercept

- *Parallel* lines have the same slope.

- *Perpendicular* lines have slopes that are negative reciprocals.

$m = \frac{1}{2}$ $m = -\frac{2}{1}$

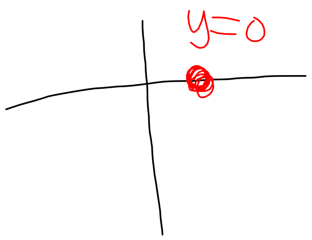


$\Delta =$ change

- Given two points, find slope using

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- *Same x-int* means find the *x*-int by substituting *y* = 0, then use this point, (*x*,0), as a point on the line.



(x, y)

- Can use any point on the line to substitute along with *m* to find b.

1. Find the equations of the following lines:

$y = mx + b$

a) passes through (4,2) with a slope of 3

m	b
$m = 3$	Need a point (4,2) x, y
	$y = 3x + b$
	$2 = 3(4) + b$
	$2 = 12 + b$
	$2 - 12 = b$
	$-10 = b$

$\therefore y = 3x - 10$

b) passes through C(3,-4) and D(-1,7)

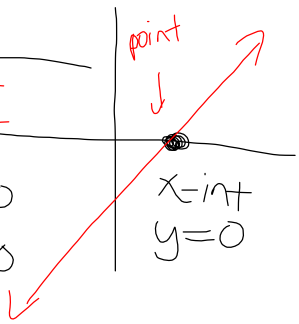
(x_1, y_1) x_2 y_2

m	b
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$y = -\frac{11}{4}x + b$ *need a point
$= \frac{7-(-4)}{-1-3}$	$-4 = -\frac{11}{4}\left(\frac{3}{1}\right) + b$
$= -\frac{11}{4}$	$\frac{-4}{1} = \frac{-33}{4} + b$
	$-\frac{16}{4} + \frac{33}{4} = b \rightarrow \frac{17}{4} = b$

$\therefore y = -\frac{11}{4}x + \frac{17}{4}$

c) perpendicular to $4x + 3y - 7 = 0$ with the same x-intercept as $2x + 3y - 12 = 0$

m	b
$4x + 3y - 7 = 0$	*Need a point
$\frac{3y}{3} = \frac{-4x}{3} + \frac{7}{3}$	$\rightarrow x$-intercept
$y = -\frac{4}{3}x + \frac{7}{3}$	$2x + 3y - 12 = 0$
$m = -\frac{4}{3}$	$2x + 3(0) - 12 = 0$
$m_\perp = \frac{3}{4}$	$2x = 12$
perpendicular	$x = 6$
	$(6, 0)$ x y



$y = \frac{3}{4}x + b$

$0 = \frac{3}{\cancel{4}_2}(\cancel{6}^3) + b$

$0 = \frac{9}{2} + b$

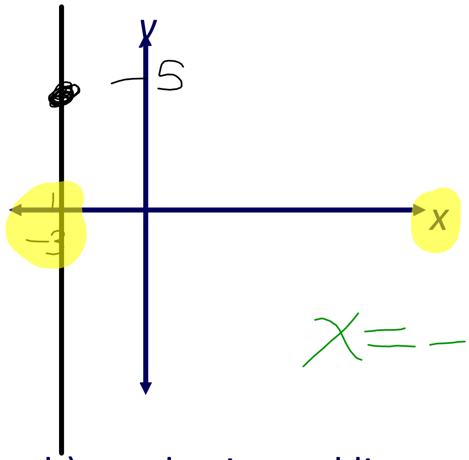
$-\frac{9}{2} = b$

$\boxed{\therefore y = \frac{3}{4}x - \frac{9}{2}}$

SPECIAL CASES: Horizontal & Vertical Lines

2. Determine the equation of each of the following lines.

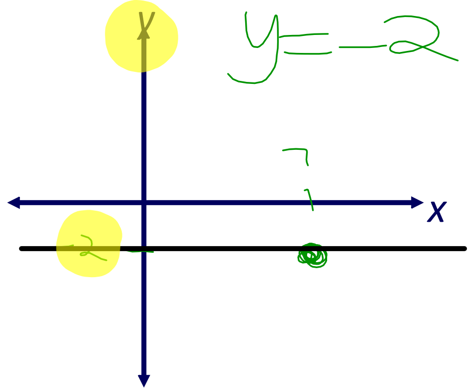
a) a vertical line passing through (-3,5)



the x-coordinate is always -3

$x = -3$

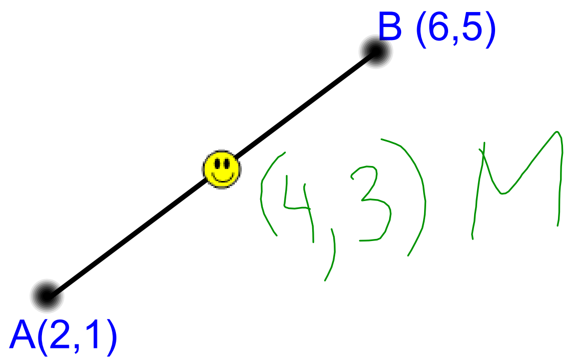
b) a horizontal line passing through (7,-2)

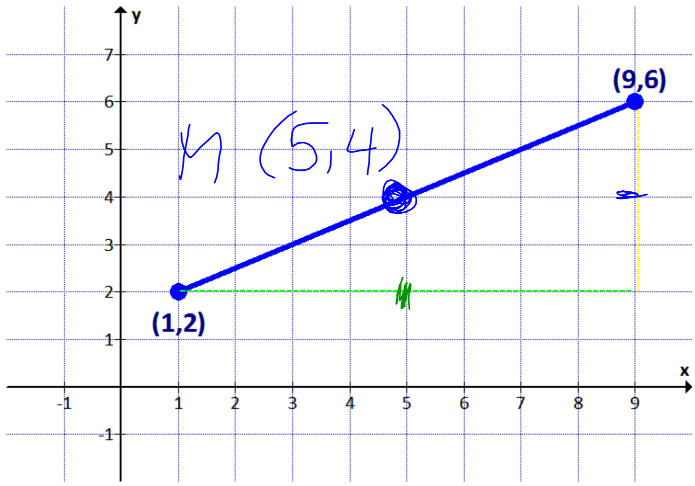


$y = -2$

the y-coordinate is always -2

The Midpoint

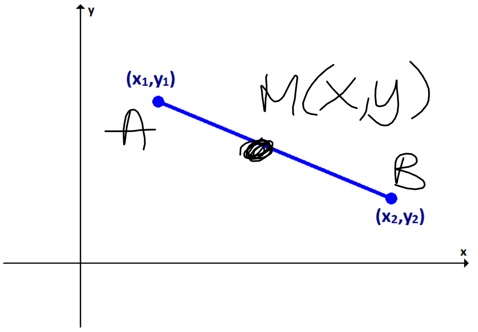




How can you determine the midpoint <u>algebraically</u> given the coordinates of the endpoint?

$$x_M = \frac{1+9}{2} = 5$$

$$y_m = \frac{2+6}{2} = 4$$



The coordinates of the midpoint of a line segment are the means of the endpoint coordinates.

aka: average

$$\boxed{M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)}$$

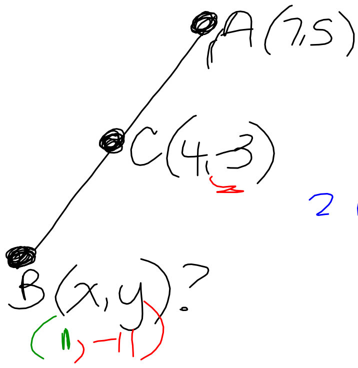
Ex. 1 Find the midpoint of the line segment AB where A(2,-4) and B(-3,5).

$$M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{2+(-3)}{2}, \frac{-4+5}{2} \right)$$

$$= \left(-\frac{1}{2}, \frac{1}{2} \right)$$

Ex. 2 C(4, -3) is the midpoint of a line segment with endpoints A(7, 5) and B. Determine the coordinates of B.



$$M(x,y) = \left(\boxed{\frac{x_1+x_2}{2}}, \frac{y_1+y_2}{2}\right)$$

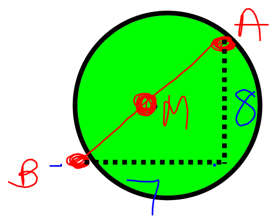
$$x_M = \frac{x_1+x_2}{2}$$
$$2(4) = \left(\frac{7+x_2}{\cancel{2}}\right)\cancel{2}$$
$$8 = 7+x_2$$
$$1 = x_2$$

$$y_M = \frac{y_1+y_2}{2}$$
$$-3 = \frac{5+y_2}{2}$$
$$-6 = 5+y_2$$
$$-11 = y_2$$

$\therefore B(1,-11)$

Ex.3 The diameter of a circle has endpoints A(4, -3) and B (-3, 5).

a) Find the centre of the circle.



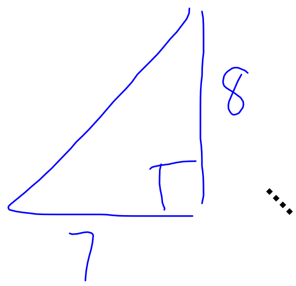
$$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{4+(-3)}{2}, \frac{(-3)+5}{2}\right)$$
$$= \left(\frac{1}{2}, 1\right)$$

$\therefore$ Center of circle is $\left(\frac{1}{2}, 1\right)$

b) Determine the diameter of the circle.

$y_2 - y_1$
$5-(-3)$
$=8$

$x_2 - x_1$
$=-3-4$
$=-7$



$$a^2+b^2=c^2$$
$$(7)^2+8^2=c^2$$
$$49+64=c^2$$
$$113=c^2$$
$$\pm\sqrt{113}=c$$
$$10.6 \doteq c, \quad c>0$$

P.55
P.66