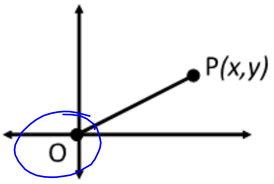
2.6 Equation of a Circle

Recall: The distance from a point *(x,y)* to the origin $(0,0)$ is:

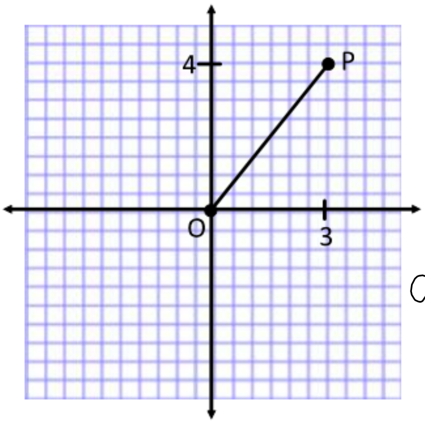


$$d_{OP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (y-0)^2}$$

$$d = \sqrt{x^2 + y^2}$$

To determine the distance from P(3,4) to the origin:



$$d_{OP} = \sqrt{x^2 + y^2}$$

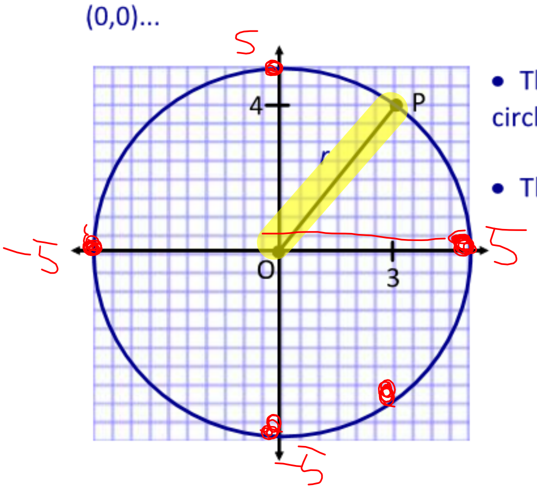
$$= \sqrt{(3)^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

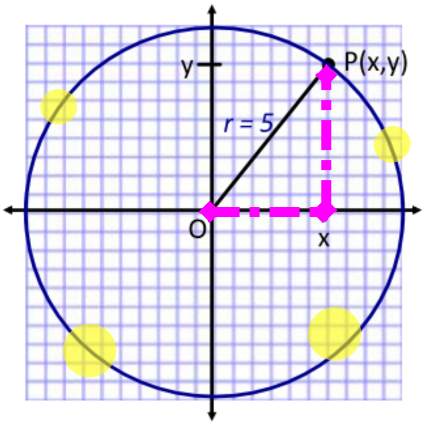
$$d_{OP} = 5$$

Now suppose the point P(3,4) is on a circle whose centre is at the origin (0,0)...



- The line segment PO is the radius of the circle.
- The circle ∴ has a radius of 5 units.

Consider the general point P(x, y) on the circle whose centre is at the origin (0,0) and whose radius is 5...



What is true for all points on the circle?

→ distance is 5 units from origin

Using the distance formula, we can now write the equation of the circle:

$d = \sqrt{x^2+y^2}$

$5 = \sqrt{x^2+y^2}$

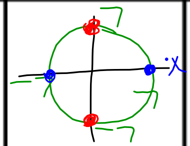
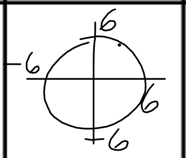
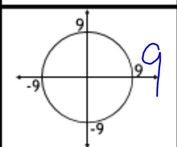
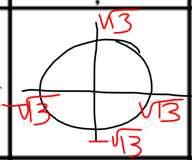
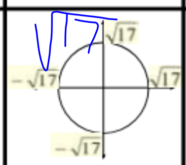
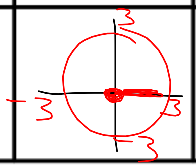
or squaring both sides:

$(r)^2$ $\boxed{25 = x^2+y^2}$

In general, the equation of a circle with centre (0,0) and radius, *r*, is given by

$$\boxed{x^2+y^2=r^2}$$

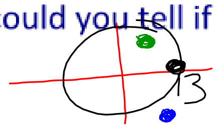
Ex. 1: Complete the table.

equation	centre	radius	sketch	x-int, y-int
$x^2+y^2=49$ r^2	$(0,0)$	$\sqrt{49}$ $=7$		$x=7$, $x=-7$ $y=7$, $y=-7$
$x^2+y^2=36$ r^2	(0, 0)	6		$x=\pm 6$ $y=\pm 6$
$x^2+y^2=81$	$(0,0)$	9		$x=\pm 9$ $y=\pm 9$
$x^2+y^2=13$ r^2	$(0,0)$	$r^2=13$ $r=\sqrt{13}$		$x=\pm\sqrt{13}$ $y=\pm\sqrt{13}$
$x^2+y^2=r^2$ $x^2+y^2=(\sqrt{7})^2$ $x^2+y^2=7$	(0, 0)	$\sqrt{7}$		$x=\pm\sqrt{7}$ $y=\pm\sqrt{7}$
$x^2+y^2=17$	$(0,0)$	$\sqrt{17}$		$x=\pm\sqrt{17}$ $y=\pm\sqrt{17}$
$x^2+y^2=9$	(0, 0)	3		±3

Ex. 2 Consider the circle $x^2 + y^2 = 169$. $r = \sqrt{169}$ $r = 13$

How could you tell if a given point $P(x,y)$ is:

a. on the circle
b. inside the circle
c. outside the circle?



a. If $\boldsymbol{x^2 + y^2 = 169}$, then the point is on the circle.
(The point satisfies the equation). (x, y)

LS	RS

$\boxed{LS = RS}$

b. If $\boldsymbol{x^2 + y^2 < 169}$, then the point is inside the circle.
(The length of line segment PO is shorter than the radius).

LS	RS

$LS < RS$

c. If $\boldsymbol{x^2 + y^2 > 169}$, then the point is outside the circle.
(The length of line segment PO is longer than the radius).

LS	RS

$LS > RS$

Ex. 3 Determine whether the following points are on, inside, or outside the circle defined by the equation $\boldsymbol{x^2 + y^2 = 169}$.

a. (-5,12)

(x, y)

LS	RS
$x^2 + y^2$	169
$= (-5)^2 + (12)^2$	
$= 25 + 144$	
$= 169$	

$\therefore (-5, 12)$ is <u>on</u> the circle

b. (11,-4)

LS	RS
$(11)^2 + (-4)^2$	169
$= 121 + 16$	
$= 137$	

LS < RS

INSIDE

c. (10,11)

LS	RS
$10^2 + 11^2$	169
$= 100 + 121$	
$= 221$	

LS > RS

$\therefore$ OUTSIDE

p 96 #1ce, 2bc, 4d, 6, 7, 8, 11bcd, 15

2.5 Handout - 2 correct

a, e

Basic: Pg. 96 #1de,2bc,6
Regular: Pg. 97 #4d,8,11bcd
Challenge: Pg. 98 #15,17

Note: Chord of a circle is a line segmen
joining 2 points on the circle.