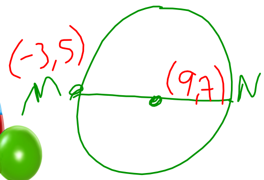
2.5 Problems: Slope, Length and Midpoint

Put it all together now.....

Ex.1 Determine the radius of a circle with endpoints of a diameter M(-3,5) and N(9,7).



$L_{MN} = \sqrt{(-3-9)^2 + (5-7)^2}$

$= \sqrt{(-12)^2 + (-2)^2}$

$= \sqrt{144+4}$

$= \sqrt{148}$

$\doteq 12.2$

$r = 12.2 \div 2$

$= 6.1$

OR

$M_{MN} = \left(\frac{-3+9}{2}, \frac{5+7}{2}\right)$

$C = (3, 6)$

$L_{CN} = \sqrt{(9-3)^2 + (7-6)^2}$

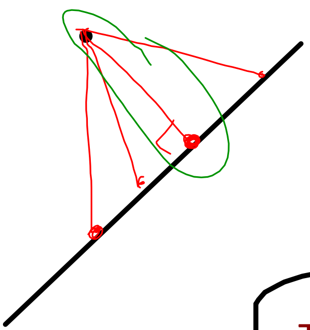
$= \sqrt{6^2 + 1^2}$

$= \sqrt{37}$

$\doteq 6.1$

Investigate!

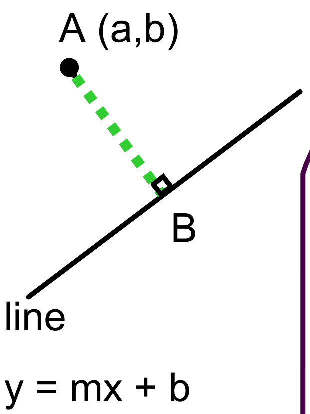
What is the shortest distance from the point to the line?



- Draw a line and a point.
- Connect the point and line with several line segments.
- Measure the line segments.
- Which is the shortest? What are its properties?

The shortest distance from a point to a line is always the length of the segment that is perpendicular to the line.

Outline a PLAN to find the distance from A to B.



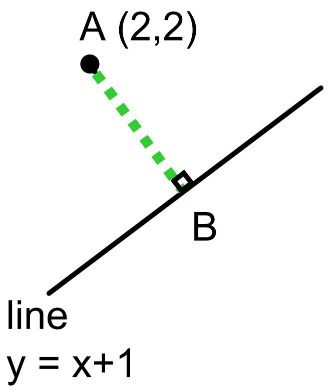
- we know the equation of the line and the coordinates of point A.

Plan:

1. slope of line, then slope of AB
2. equation of line AB
   (using slope and point A)
3. elimination/substitution to find intersection of AB and line (coordinates of B)
4. distance formula to find length of AB

recall: $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex. 2 Find the shortest distance from (2,2) to the line $y = x+1$



① $m = 1, \quad m_{\perp} = -1$

② $y = -x + b$

sub (2,2)

$2 = -2 + b$

$4 = b$

$\boxed{\therefore y = -x + 4}$

③ $y = x + 1$ ①

$y = -x + 4$ ②

ADD

$2y = 5$

$y = \frac{5}{2}$

sub into ①

$\frac{5}{2} = x + 1$

$\frac{5}{2} - 1 = x$

$\frac{3}{2} = x$

$\therefore (x, y) = (\frac{3}{2}, \frac{5}{2})$

④ $(\frac{3}{2}, \frac{5}{2})\ (2,2)$

$L = \sqrt{(2 - \frac{3}{2})^2 + (2 - \frac{5}{2})^2}$

$= \sqrt{(\frac{1}{2})^2 + (\frac{-1}{2})^2}$

$= \sqrt{\frac{1}{4} + \frac{1}{4}}$

$= \sqrt{\frac{1}{2}}$

$\doteq 0.71$

$\therefore$ The shortest distance is 0.7 units

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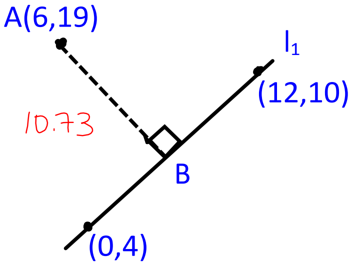
Ex.3 Given the line containing the point (0,4) and (12,10), determine the distance from A(6,19) to the line.



Equation of l_1:

$m = \frac{10-4}{12-0} = \frac{6}{12} = \frac{1}{2}$

$y = \frac{1}{2}x + b$

sub (0,4)

$4 = \frac{1}{2}(0) + b$

$4 = b$

$\boxed{\therefore y = \frac{1}{2}x + 4}$ ✓✓

Equation of AB

$m_{\perp} = -2$

$y = -2x + b$

sub A(6,19)

$19 = -2(6) + b$

$19 + 12 = b$

$31 = b$

$\boxed{\therefore y = -2x + 31}$ ✓✓

POI:

$y = \frac{1}{2}x + 4$ ①

$y = -2x + 31$ ②

Sub ① into ②

$(2)\ \frac{1}{2}x + 4 = -2x + 31\ (2)$

$x + 8 = -4x + 62$

$5x = 54$

$\boxed{x = \frac{54}{5}}$

sub in ①

$y = \frac{1}{2}\left(\frac{54}{5}\right) + 4$

$y = \frac{54}{10} + \frac{40}{10}$

$y = \frac{94}{10}$

$y = \frac{47}{5}$

(2 - 4) ✓

$\therefore (x,y) = \left(\frac{54}{5}, \frac{47}{5}\right)$

Length: $\left(\frac{54}{5}, \frac{47}{5}\right)$ (6,19)

$L = \sqrt{\left(\frac{54}{5}-6\right)^2 + \left(\frac{47}{5}-19\right)^2}$

$= \sqrt{\left(\frac{54}{5}-\frac{30}{5}\right)^2 + \left(\frac{47}{5}-\frac{95}{5}\right)^2}$

$= \sqrt{\left(\frac{24}{5}\right)^2 + \left(\frac{-48}{5}\right)^2}$ ✓✓

$\doteq 10.73$