2^5

3.7 Negative and Zero Exponents

Ex. 1 Complete the table of values for $y = 2^x$. Graph $y = 2^x$.

x	$y=2^x$
5	32
4	16
3	8
2	4
1	2
0	1

÷2 ÷2 ÷2 ÷2 ÷2

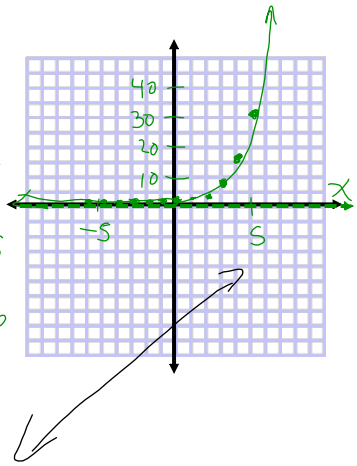
2^0

no decimals...use fractions

x	$y=2^x$
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$
-3	$\frac{1}{8}$
-4	$\frac{1}{16}$
-5	$\frac{1}{32}$
-6	$\frac{1}{64}$

÷2 ÷2 ÷2 ÷2 ÷2 ÷2

$2^{-2} = \frac{1}{4}$ $2^{-3} = \frac{1}{8}$ $2^{-4} = \frac{1}{16}$



Describe the graph. How does it compare to $y = x^2$?

$y = 2^x$

→ increasing

→ non-linear

→ increasing slowly then more quickly

→ graph will never touch x-axis ($y \neq 0$) [asymptote at $y=0$] 3u

Compare

$y=2^x$	$y=x^2$
non-linear	
increasing	increases/decreases decreases/increases
both have y-intercepts	
no x-int	can have x-int
not	Symmetrical
infinite # of points	

What x-can be: all real numbers

RANGE What y-can be: $y>0$ min/max determines y-values $y \geq 0$

Will the graph ever cross the x-axis? Explain.

$y = 2^x$ No → approaches the x-axis

can't divide something into nothing ($y \neq 0$)

Ex. 2 Complete the table for $y = 3^x$.

no decimals...use fractions

x	$y=3^x$
5	243
4	81
3	27
2	9
1	3
0	1

÷3 ÷3 ÷3 ÷3 ÷3

x	$y=3^x$
-1	$\frac{1}{3}$
-2	$\frac{1}{9}$
-3	$\frac{1}{27}$
-4	$\frac{1}{81}$
-5	$\frac{1}{243}$
-6	$\frac{1}{729}$

Think about what a negative exponent means!

3^1

$3^0 = 1$

$2^0 = 1$

$$\frac{3^{-4}}{1} = \left(\frac{1}{3}\right)^4$$

Ex. 3 Use the pattern in the previous examples to determine the value of:

a) 4^{-1}

$= \left(\frac{1}{4}\right)^{1}$

$= \frac{1}{4}$

b) 5^{-2}

$= \left(\frac{1}{5}\right)^{2}$

$= \frac{1}{25}$

c) 7^{-3}

$= \left(\frac{1}{7}\right)^{3}$

$= \frac{1}{343}$

d) 4^{-2}

$= \left(\frac{1}{4}\right)^{2}$

$= \frac{1}{16}$

e) 5^{0}

$= 1$

f) 4^{0}

$= 1$

g) 9^{0}

$= 1$

h) 435^{0}

$= 1$

Rule: for any non-zero base "a"

$a^{0} = 1$ and $a^{-k} = \frac{1}{a^{k}}$

Ex. 4 Evaluate. No decimals.

a) 2^{-3}

$= \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$

b) 3^{-4}

$= \left(\frac{1}{3}\right)^4$

$= \frac{1}{81}$

c) 5^{-3}

$= \left(\frac{1}{5}\right)^3$

$= \frac{1}{125}$

d) 6^{-2}

$= \left(\frac{1}{6}\right)^2$

$= \frac{1}{36}$

e) $(-2)^{-4}$

$= \left(-\frac{1}{2}\right)^{*4}$

$= \frac{1}{16}$

f) $(-3)^{-1}$

$= \left(-\frac{1}{3}\right)^1$

$= -\frac{1}{3}$

g) $(-4)^{-3}$

$= \left(-\frac{1}{4}\right)^3$

$= -\frac{1}{64}$

h) -5^{-2}

$= -\left(\frac{1}{5}\right)^2$

$= -\frac{1}{25}$

Base (−)

→ even exponent (+)

→ odd exp (−)

Ex. 5 Evaluate. No decimals.

a) $\left(\frac{1}{4}\right)^{-2}$

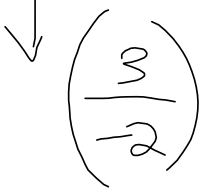
$= \left(\frac{4}{1}\right)^{2}$

$= 16$

b) $\left(\frac{-2}{3}\right)^{-3}$

$= \left(-\frac{3}{2}\right)^{3}$

$= -\frac{27}{8}$



c) $\left(\frac{-1}{5}\right)^{-1}$

$= \left(\frac{5}{-1}\right)^{1}$

$= -5$

d) $\left(\frac{4}{3}\right)^{-2}$

$= \left(\frac{3}{4}\right)^{2}$

$= \frac{9}{16}$

Ex. 6 A bacteria colony decays by ½ of its original population every 5 hours.

a) What fraction remains after 20 hours? 30 hours? 50 hours? Positive Exponent

time	0	5	10	15	20	30	50
fraction left	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{1024}$
	$(\frac{1}{2})^0$	$(\frac{1}{2})^1$	$(\frac{1}{2})^2$	$(\frac{1}{2})^3$	$(\frac{1}{2})^4$	$(\frac{1}{2})^6$	$(\frac{1}{2})^{10}$

1 2 3 4 6 10

20 hours → halved 4 times ∴ $(\frac{1}{2})^4$

30 → $(\frac{1}{2})^6$

50 → $(\frac{1}{2})^{10}$

b) Write each fraction from a) as a power with a negative exponent. Negative Exponen

20 hours $(\frac{1}{2})^4 = 2^{-4}$

30 hours: 2^{-6}

50 hours: 2^{-10}

c) If the colony started with 32768 bacteria. How many remain after 25 hours?

$32\,768\left(\frac{1}{2}\right)^5$ *

$= 32768\left(\frac{1}{32}\right)$

$(= 1)$

whatever megan says.

$\frac{1}{2}$ 5x

32 768 → 5 hours → 16 → 10 h → 8 → 15

p. 199

Start Review.