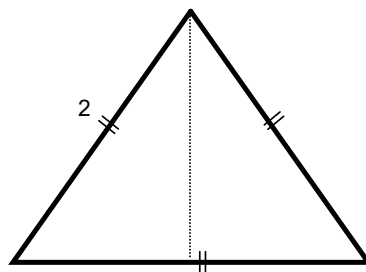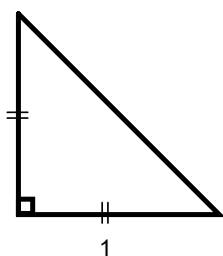# Lesson 4.2A : Special Angles and the Unit Circle

Consider the following triangles.
Determine the measure of all of the sides and the angles.

These are the Special Triangles:

45^0 (Right Isosceles Triangle)

$\sin 45^0 =$

$\cos 45^0 =$

$\tan 45^0 =$

$30^0, 60^0$ (Half an Equilateral Triangle)

$\sin 30^0 =$ $\sin 60^0 =$

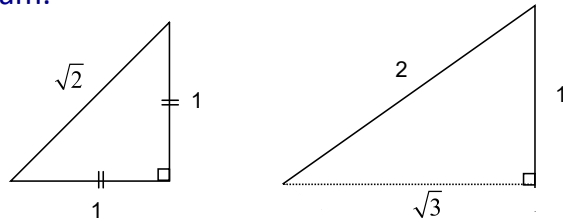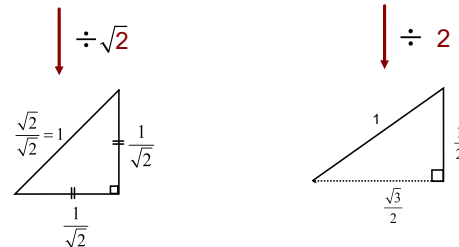$\cos 30^0 =$ $\cos 60^0 =$

$\tan 30^0 =$ $\tan 60^0 =$

The Unit Circle: The unit circle is a way to "standardize" the ratios of the special angles onto one diagram.
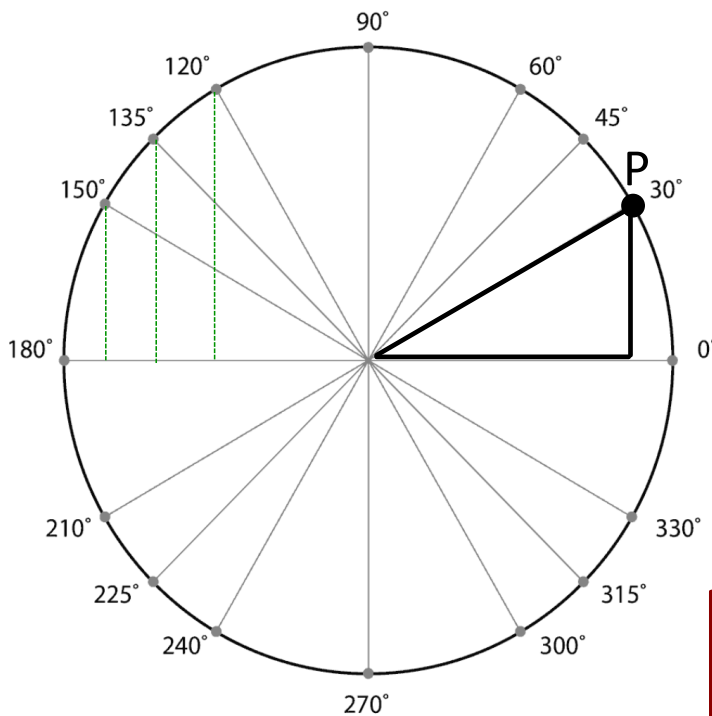
The original special triangles:

$\sqrt{2}$ 1   2   1

1      $\sqrt{3}$

$\div \sqrt{2}$ $\div 2$

Now, make the hypotenuse 1:

$\frac{\sqrt{2}}{\sqrt{2}} = 1$ $\frac{1}{\sqrt{2}}$ 1 $\frac{1}{2}$

$\frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2}$

Note: These are similar triangles to the original triangles.

Consider these triangles on a circle with radius of 1 (the terminal arm is 1 unit long).

- Label the lengths of the sides.
- What are the coordinates of point P?

- What are the values of the primary trig ratios?
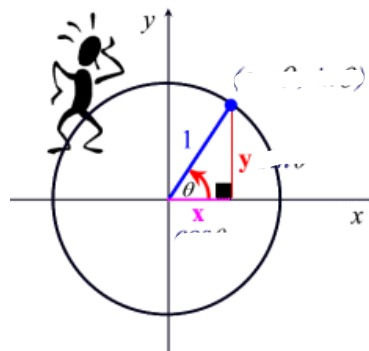
- What can you conclude?
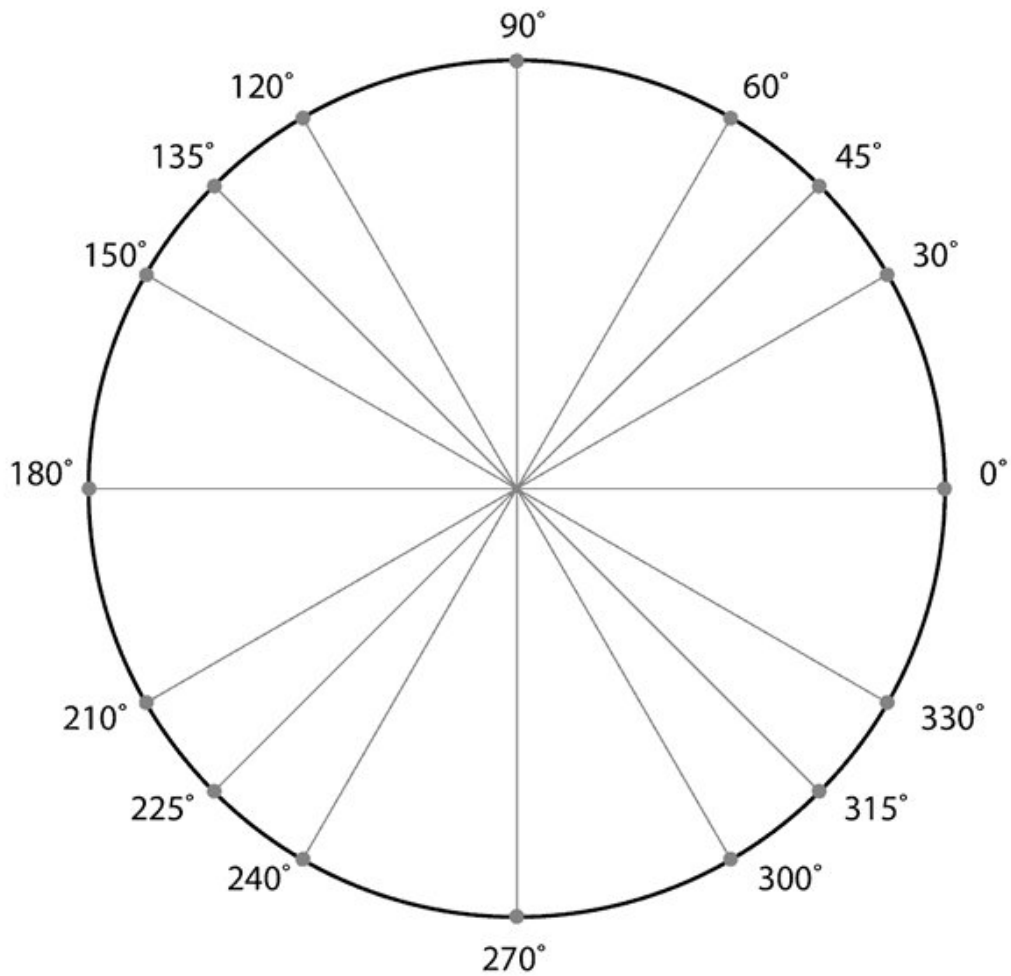
The coordinates are (c   °,   °).

In general:

$$\cos \theta = \frac{x}{1} \qquad \sin \theta = \frac{y}{1} \qquad \tan \theta = \frac{y}{x}$$
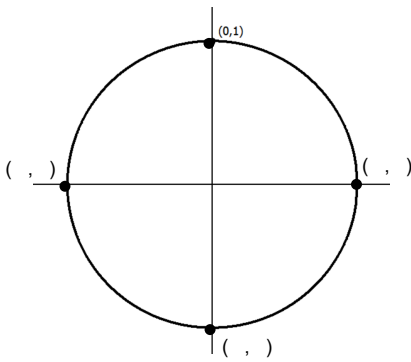
Creating the Unit Circle

Think about the following...
- What happens to the y values as you rotate?
- What happens to the x values as you rotate?
- Connect the CAST rule to your knowledge of reflecting in the x-axis or y-axis.

The unit circle allows us to understand the values of trig ratios for <u>axis angles</u>.

Terminal arm lies on the x-axis or y-axis

	0°	90°	180°	270°	360°
sin θ					
cos θ					
tan θ					

Ex. 1   Determine the exact values.

a) $\cos 60^0$

b) $\sin 45^0$

c) $\tan 30^0$

d) $\sin 240^0$

e) $\tan 135^0$

f) $\sin 150^0$

g) $\tan\left(-300^0\right)$

.

Ex. 2   Determine all possible values for $0 < \theta < 360^0$.

a) $\sin \theta = \dfrac{1}{\sqrt{2}}$

b) $\tan \theta = -\dfrac{1}{\sqrt{3}}$

c) $\sin \theta = \dfrac{-\sqrt{3}}{2}$

d) $\sin \theta = -1$

e) $\tan \theta = undefined$

Pull

Ex 3:   Evaluate the following using exact values.

$$\sin 30^0 \cos^2\left(225^0\right) - \tan\left(-60^0\right)$$