# Lesson 4.6B - Trig Identities (Day 2)

## Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\csc^2\theta = \frac{1}{\sin^2\theta} \qquad \sec^2\theta = \frac{1}{\cos^2\theta} \qquad \cot^2\theta = \frac{1}{\tan^2\theta}$$

## Quotient Identities

☆ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \qquad \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$$

## Pythagorean Identities

☆ $\sin^2\theta + \cos^2\theta = 1$

*These can be rearranged.*

$$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

## Strategies

STEPS TO PROVING IDENTITIES


1. Separate LS from RS.
2. Write both sides in terms of sin x and cos x.
3. To make LS = RS , try :
   - Factoring.
   - Simplifying.
   - Substitute any of the identities we just learned.
   - In some situations, multiply by the conjugate.

### Factoring

$1-\cos^2\theta$

$\sin x-\sin^2 x$

$\sin^2\theta-2\sin\theta+1$

$\sin^2\theta-\cos^2\theta$

$\cos^2\theta-7\cos\theta+10$

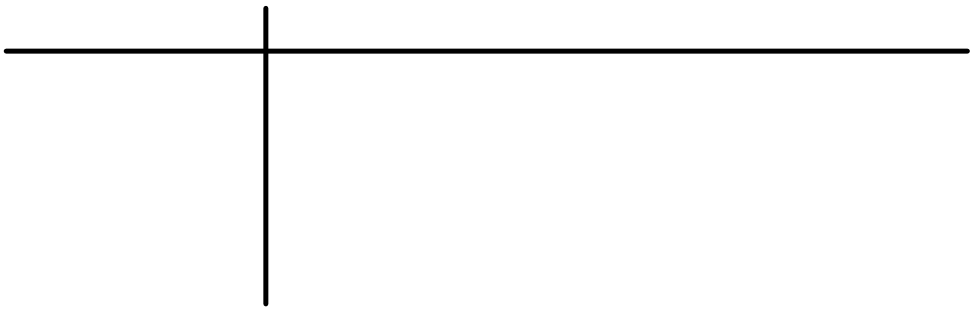
$6\sin^2\theta-\sin\theta-1$

### Multiplying by the conjugate

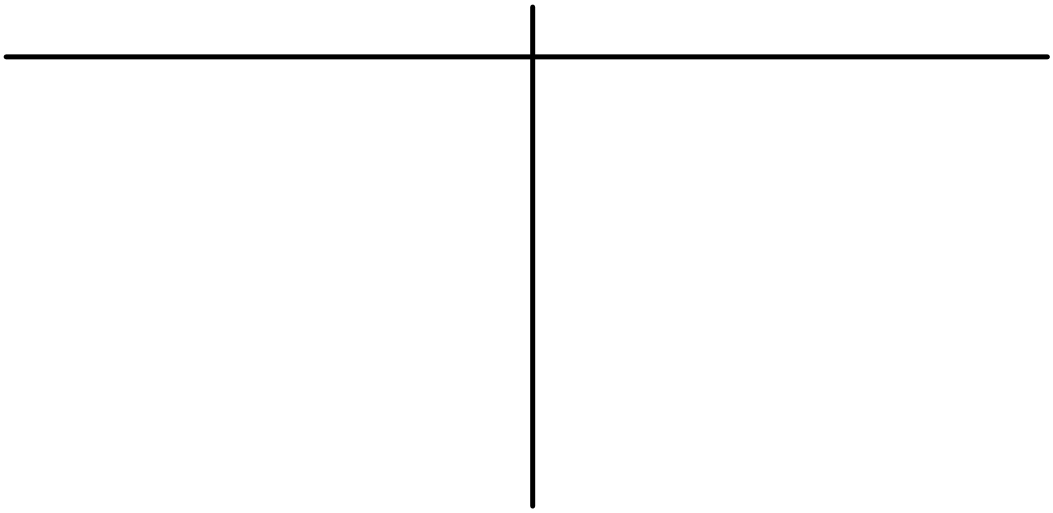
$$\frac{1}{1-\cos x}$$

Examples - Prove the following identities.

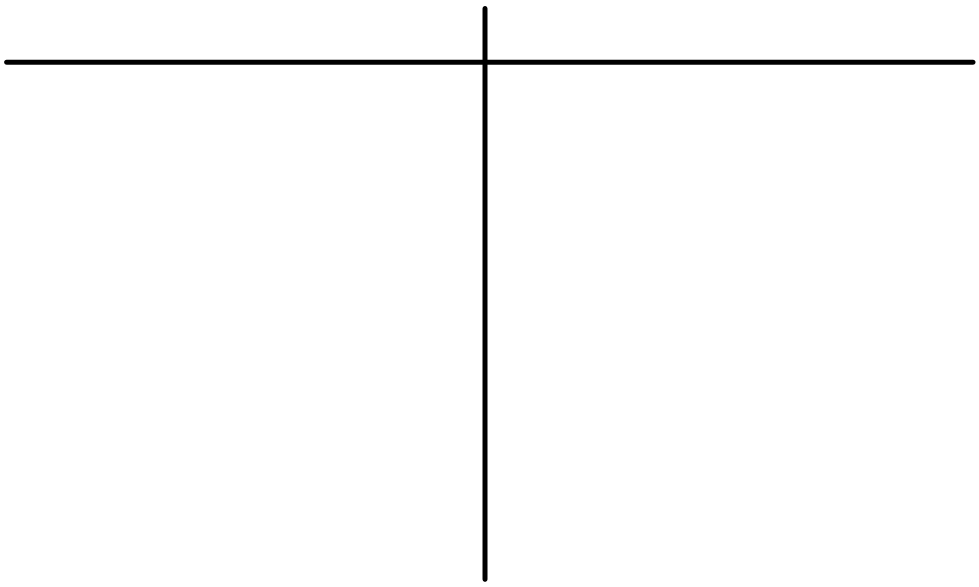
a) $\dfrac{1}{\cot x} = \sin x \sec x$



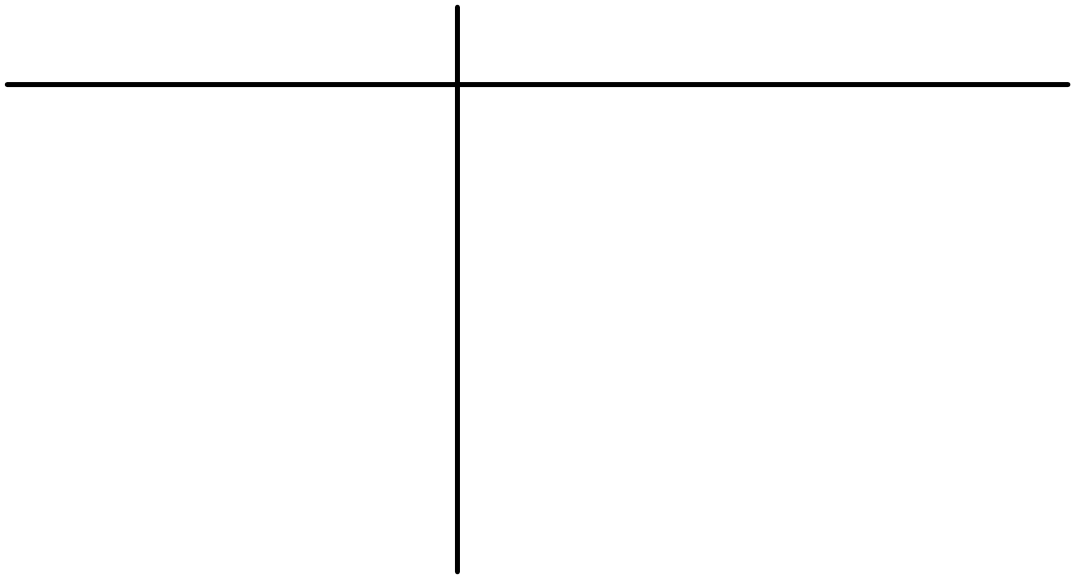
b) $\dfrac{1+\cot x}{\csc x} = \sin x + \cos x$



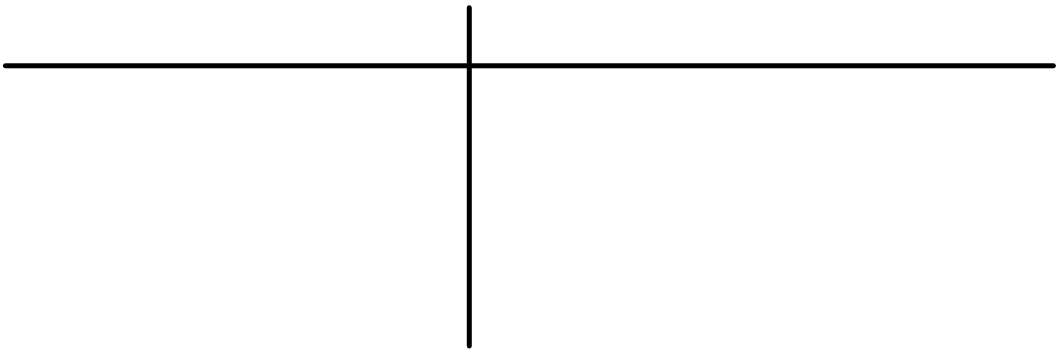
c) $\dfrac{\cos\theta - 1}{1 - \sec\theta} = \dfrac{\cos\theta + 1}{1 + \sec\theta}$



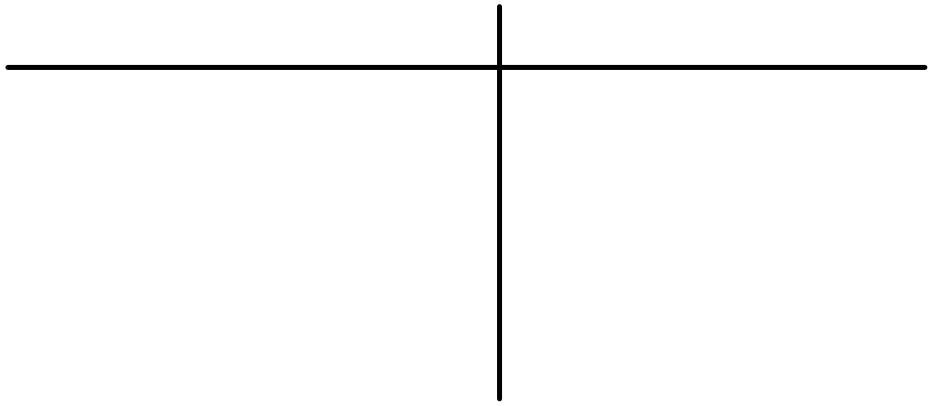
d) $\tan\alpha \sin\alpha + \cos\alpha = \sec\alpha$



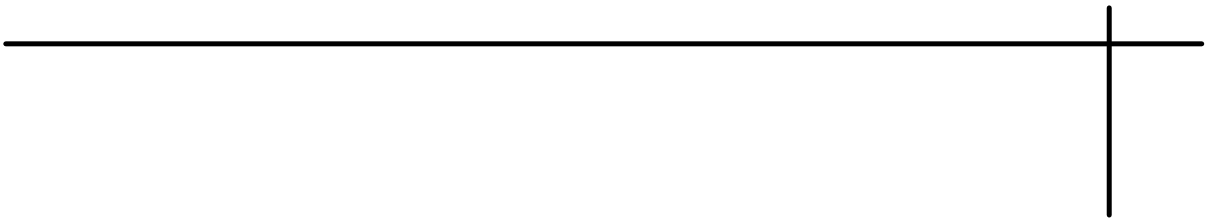
e) $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$



f) $\sin x - \sin x \cos^2 x = \sin^3 x$



g) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$



h) $\dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \dfrac{3 + \sin x}{1 - \sin x}$