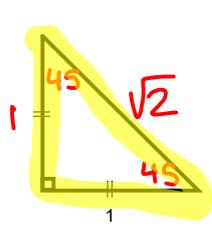


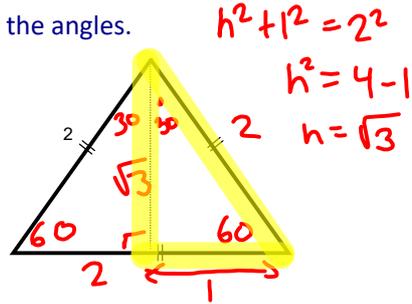
Lesson 4.2 : Special Angles and the Unit Circle

Consider the following triangles.

Determine the measure of all of the sides and the angles.



$$1^2 + 1^2 = c^2$$



$$h^2 + 1^2 = 2^2$$

$$h^2 = 4 - 1$$

$$h = \sqrt{3}$$

These are the Special Triangles:

45° (Isosceles Triangle)

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1$$

30°, 60° (Half an Equilateral Triangle)

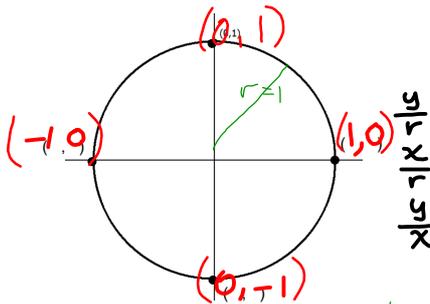
$$\sin 30^\circ = \frac{1}{2} \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \tan 60^\circ = \sqrt{3}$$

Now consider the Axis Angles (the terminal arm lies on the x-axis or y-axis)

Complete the table for each of the axis angles given the following diagram.

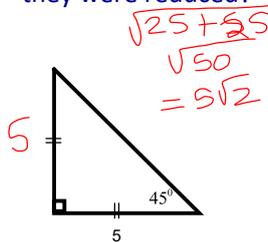


x ו y קואורדינטות

	0°	90°	180°	270°	360°
sin θ	0	1	0	-1	0
cos θ	1	0	-1	0	1
tan θ	0	undef	0	undef	0
tan θ	1	-1	1	-1	



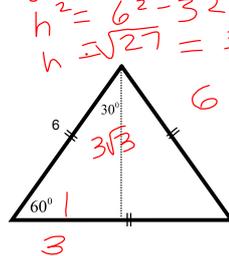
What happens if the above diagrams are enlarged? Would it be the same result if they were reduced? * these triangles are "similar" to the above triangles



$$\sqrt{25 + 25}$$

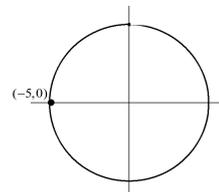
$$\sqrt{50}$$

$$= 5\sqrt{2}$$



$$h^2 = 6^2 - 3^2$$

$$h = \sqrt{27} = 3\sqrt{3}$$



$$\sin 45^\circ = \frac{5}{5\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\sin 60^\circ = \frac{3\sqrt{3}}{6}$$

$$= \frac{\sqrt{3}}{2}$$

$$\sin 270^\circ =$$

Ex. 1 Determine the exact values, if possible, of each trig ratio.

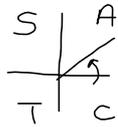
Steps:

1. Determine the quadrant.
2. Diagram & find related angle. θ_r
3. Use special angles or the unit circle, if appropriate.
4. Use CAST rule to determine sign (+ or -).

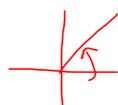
$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$$

Pull

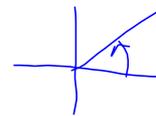
a) $\cos 60^\circ = +\frac{1}{2}$



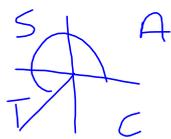
b) $\sin 45^\circ = +\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



c) $\tan 30^\circ = +\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

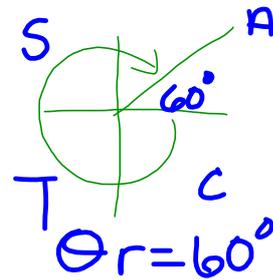


d) $\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

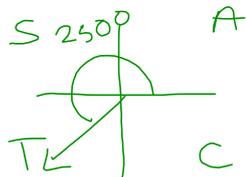


$\theta_r = 240 - 180 = 60^\circ$

e) $\tan(-300^\circ) = +\tan 60^\circ = \sqrt{3}$

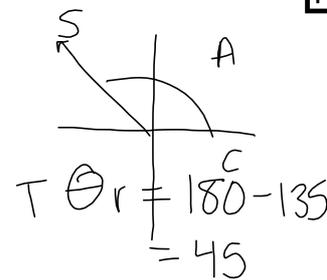


$\cos 250^\circ = -\cos 70^\circ$
Use calc
 ≈ -0.342



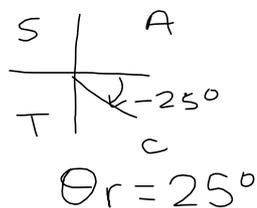
$\theta_r = 250 - 180 = 70^\circ$

$\tan 135^\circ = -\tan 45^\circ = -1$

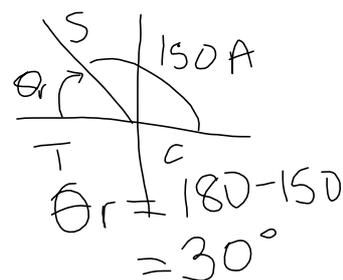
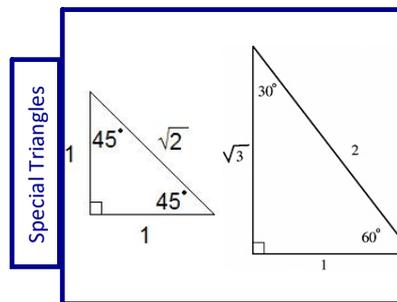


The Unit Circle

$\cos(-25^\circ) = \cos 25^\circ \approx 0.906$



$\sin 150^\circ = +\sin 30^\circ = \frac{1}{2}$



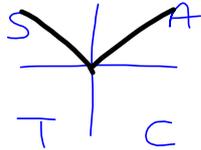
Ex. 2 Determine all possible values for $0 < A < 360^\circ$.

→ 2 answers!

Steps:

1. Determine the quadrants.
2. Draw a diagram with terminal arms.
3. Determine the related angle.
4. Find the principal angles.

$$\sin A = \frac{1}{\sqrt{2}}$$



$$\theta_r = 45^\circ$$

Q1

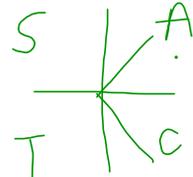
$$A = 45^\circ$$

Q2

$$A = 180 - 45 = 135^\circ$$

$$\therefore A = 45^\circ, 135^\circ$$

$$\cos A = \frac{1}{2}$$



$$\theta_r = 60^\circ$$

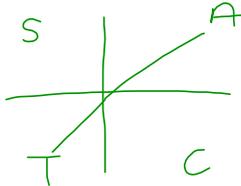
Q1

$$A = 60^\circ$$

Q4

$$A = 360 - 60 = 300^\circ$$

$$\tan A = 1$$



$$\theta_r = 45^\circ$$

Q1

$$A = 45^\circ$$

Q3

$$A = 180 + 45 = 225^\circ$$

$$\therefore A = 45^\circ, 225^\circ$$

$$\tan A = -\frac{\sqrt{3}}{1}$$

$$\theta_r = 60^\circ$$

$$A = 120^\circ, 300^\circ$$

$$\cos A = -0.9675$$

calculator

$$\theta_r = \cos^{-1}(0.9675)$$

$$\theta_r = 15^\circ$$



Q2

$$A = 180 - 15 = 165^\circ$$

Q3

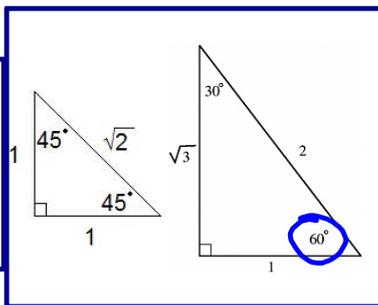
$$A = 180 + 15 = 195^\circ$$

$$\theta_r = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\theta_r = 18^\circ$$

$$A = 18^\circ, 198^\circ$$

Special Triangles



$$\tan A = \frac{1}{3}$$