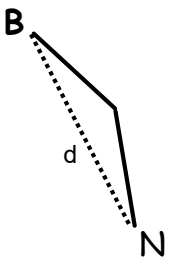
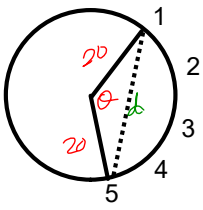
#7



#10

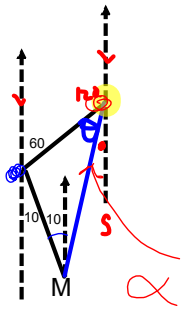


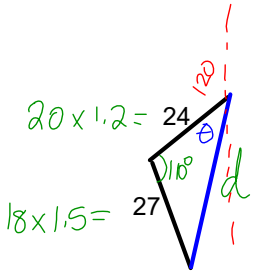
∠ in between cars
$= 360 \div 10$
$= 36°$

$\theta = 4 \times 36$
$= 144$

$d^2 = 20^2 + 20^2 - 2(20)(20)\cos 144$
$d^2 = 1447.2$
$d = 38\ m$

#11





$d^2 = 27^2 + 24^2 - 2(27)(24)\cos 110$

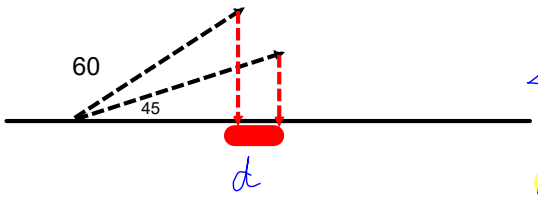
$d \doteq 42\ km$

$\frac{\sin\theta}{27} = \frac{\sin 110}{42}$

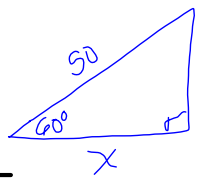
$\theta = 37°$

$\alpha = 180 - 120 - 37$
$= 23°$

23° west of South



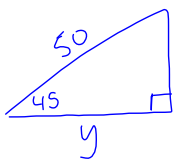
horizontal distance
$y - x$
$= 25\sqrt{2} - 25$
$= 25(\sqrt{2} - 1)$



$\cos 60 = \frac{x}{50}$

$\frac{1}{2} = \frac{x}{50}$

$25 = x$



$\cos 45 = \frac{y}{50}$

$\frac{\sqrt{2}}{2} = \frac{y}{50}$

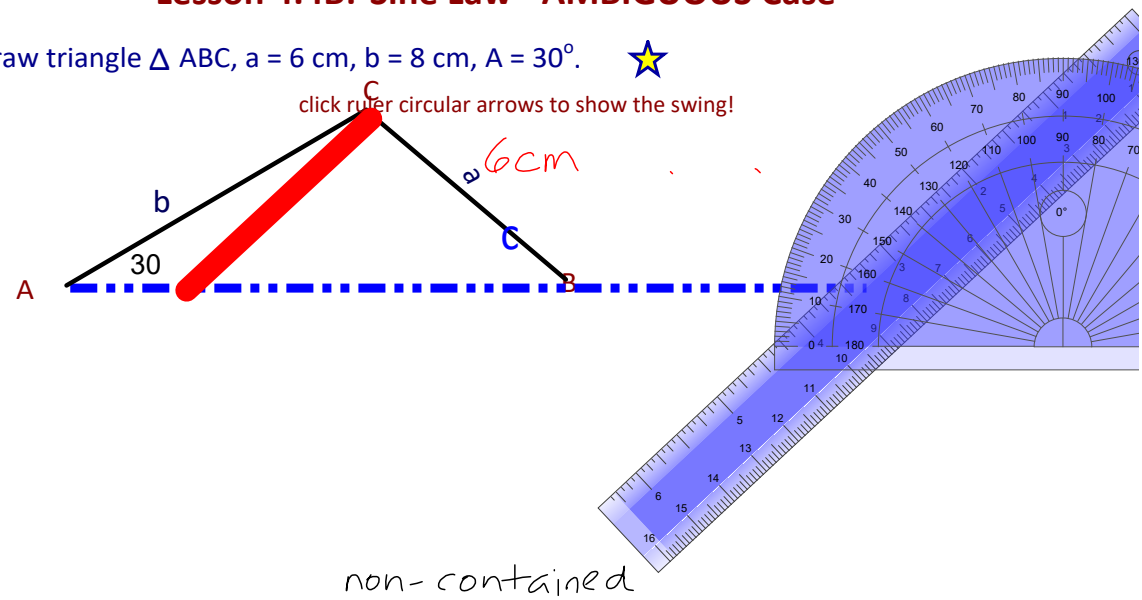
$\frac{50\sqrt{2}}{2} = y$

$25\sqrt{2} = y$

Lesson 4.4B: Sine Law - AMBIGUOUS Case

Draw triangle Δ ABC, a = 6 cm, b = 8 cm, A = 30°.

click ruler circular arrows to show the swing!

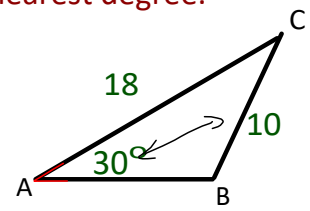


non-contained

- When two sides and the non-included angle of a triangle are given, the triangle may not be unique. (SSA)
- You will have to determine if there is no solution, one solution or two possible solutions.

Ex. 1: Given that ΔABC has <A = 30°, a = 10, and b = 18, find the value of <B to the nearest degree.

Sine Law SSA → Ambiguous Case?



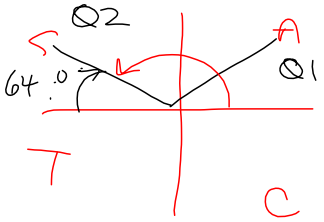
$$\frac{\sin B}{18} = \frac{\sin 30}{10}$$

$$\sin B = \frac{18 \sin 30}{10}$$

$$\sin B = 0.9$$

$$\therefore B = \sin^{-1}(0.9)$$

$$B = 64°$$



OR Q2

$B = 180 - 64$

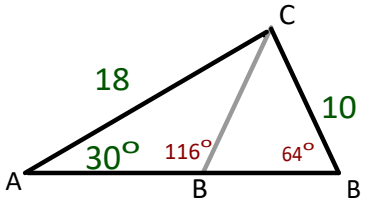
$= 116°$

$\therefore B = 64°$ OR $116°$

② ∠A = 30
∠B = 116
∠C = 34° ✓

① ∠A = 30
∠B = 64
∠C = 86° ✓

As we see algebraically, there are two possible answers to this question.



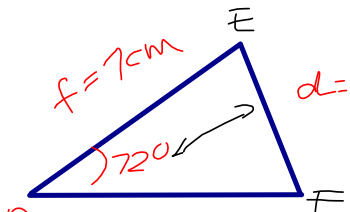
Therefore, it is very important to always *consider* both solutions (Q1 & Q2) when using Sine Law to solve a triangle given SSA.

Ex. 2: Determine the measures of all angles in the given triangles.

a) In ΔDEF, < D = 72°, d = 9 cm, f = 7 cm.

"bigger"

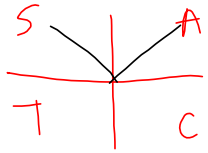
SSA → Sine Law
* Ambiguous Case?



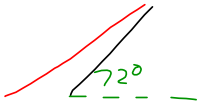
$$\frac{\sin F}{7} = \frac{\sin 72}{9}$$

$$\sin F = \frac{7\sin 72}{9}$$

$$\sin F = \boxed{0.74}$$



← +ve Sine ratio



← F ≐ 48°

E = 180 - 72 - 48
E = 60°

OR

F = 180 - 48
= 132°

72 + 132 ≥ 180
NOT GOOD
NOT POSSIBLE

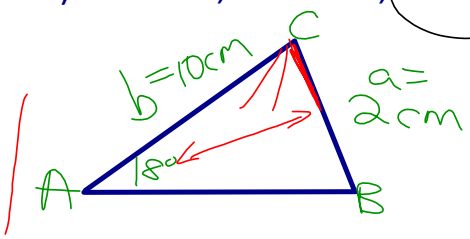
Pull

This example shows the case of one solution.
There is only 1 possible triangle that can be constructed.

Small

b) In ΔABC, < A = 18°, a = 2 cm, b = 10 cm.

Sine Law → SSA
* Ambiguous?



Pull

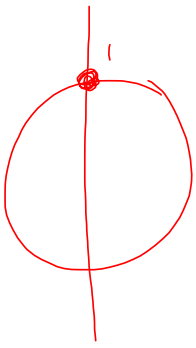
$$\frac{\sin B}{10} = \frac{\sin 18}{2}$$

$$\sin B = \frac{10\sin 18}{2}$$

$$\sin B \doteq 1.55$$

B = Impossible

Sine can't be greater than 1

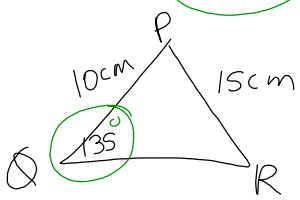


No possible triangle

c) In ΔPQR, < Q = 135°, q = 15 cm, r = 10cm (Find all angles).

obtuse angle ← 1 solution

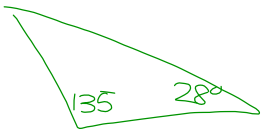
SSA
Sine Law
*Ambiguous?



$$\frac{\sin R}{10} = \frac{\sin 135}{15}$$

$$\sin R = \frac{10 \sin 135}{15}$$

$$\sin R \doteq \frac{10 \cdot \frac{\sqrt{2}}{2}}{15}$$



$R = 28°$

$P = 180 - 135 - 28$

$P = 17°$

~~Q2~~
$180 - 28 = 152°$
$135 + 152°$

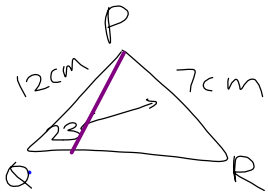
Pull

This example shows the case of one solution.
There is only 1 possible triangle that can be constructed.

d) In ΔPQR, < Q = 23°, q = 7cm, r = 12cm (SOLVE).

SSA
Sine Law
*Ambiguous case?

Pull



$$\frac{\sin R}{12} = \frac{\sin 23}{7}$$

$$\sin R = \frac{12 \sin 23}{7}$$

$$\sin R \doteq 0.67$$

$R \doteq 42°$ OR $R \doteq 180 - 42 \doteq 138°$

$P = 180 - 42 - 23 = 115°$

$P = 180 - 23 - 138 = 19°$

$$\frac{p}{\sin 115} = \frac{7}{\sin 23}$$

$$p = \frac{7 \sin 115}{\sin 23}$$

$p \doteq 16.2$ cm

$$\frac{p}{\sin 19} = \frac{7}{\sin 23}$$

$p \doteq 5.8$ cm

$\therefore R = 42°$, $P = 115°$, $p = 16.2$ OR $R = 138°$, $P = 19°$, $p = 5.8$ cm

Homework

pg 254 #6a, 14bcd, 16, 21abc

b

am•big•u•ous

doubtful, uncertain,
unclear in meaning