# Lesson 4.6B - Trig Identities (Day 2)

## Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\csc^2\theta = \frac{1}{\sin^2\theta} \qquad \sec^2\theta = \frac{1}{\cos^2\theta} \qquad \cot^2\theta = \frac{1}{\tan^2\theta}$$

## Quotient Identities

☆ $$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \qquad \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$$

## Pythagorean Identities

☆ $\sin^2\theta + \cos^2\theta = 1$ ← have to know

*These can be rearranged.*

$$\boxed{1+\tan^2\theta = \sec^2\theta} \qquad \boxed{1+\cot^2\theta = \csc^2\theta}$$

you could prove
using sinθ/cosθ

## Strategies

STEPS TO PROVING IDENTITIES

PROOF

1. Separate LS from RS.
2. Write both sides in terms of sin x and cos x.
3. To make LS = RS , try :
   - Factoring.
   - Simplifying.
   - Substitute any of the identities we just learned.
   - In some situations, multiply by the conjugate.

## Factoring

$\sin^2\theta$ → dos

$1-\cos^2\theta$

$=(1-\cos\theta)(1+\cos\theta)$

changes $\sin^2\theta$ to $\cos\theta$

$1-x^2$
$=(1-x)(1+x)$

$\sin x-\sin^2 x$

$=\sin x(1-\sin x)$

$x-x^2$
$=x(1-x)$

$1-\sin^2 x=\cos^2 x$

$\sin^2\theta-2\sin\theta+1$

$=(\sin\theta-1)(\sin\theta-1)$

$=(\sin\theta-1)^2$

$x^2-2x+1$
$=(x-1)(x-1)$
$=(x-1)^2$

M: 1
A: -2
N: -1, -1

$\sin^2\theta-\cos^2\theta$ dos

$=(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)$

perfect square

$\boxed{\sin^2\theta}+\boxed{2\sin\theta\cos\theta}+\boxed{\cos^2\theta}$

$=(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)$

$=(\sin\theta+\cos\theta)^2$

$x^2+2xy+y^2$
$=(x+y)^2$

$\cos^2\theta-7\cos\theta+10$

$=(\cos\theta-5)(\cos\theta-2)$

$x^2-7x+10$
$=(x-5)(x-2)$

$6\sin^2\theta-\sin\theta-1$

$=(3\sin\theta+1)(2\sin\theta-1)$

$6x^2-x-1$
$=(3x+1)(2x-1)$

M: -6
A: -1
N: $\frac{-3}{6x}, \frac{2}{6x}$
$=\frac{-1}{2x}\ \frac{1}{3x}$

## Multiplying by the conjugate

$$\frac{1}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}$$

$$=\frac{1+\cos x}{(1-\cos x)(1+\cos x)}$$

$$=\frac{1+\cos x}{1-\cos^2 x}$$

$\sin^2 x$

Examples - Prove the following identities.

a) $\dfrac{1}{\cot x} = \sin x \sec x$

| LS | RS |
|---|---|
| $\dfrac{1}{\cot x}$ | $\sin x \sec x$ |
| $= \dfrac{1}{\frac{\cos x}{\sin x}}$ | $= \sin x \cdot \dfrac{1}{\cos x}$ |
| $\boxed{= \tan x}$ | $= \tan x$ |
| $= \dfrac{\sin x}{\cos x}$ | $= \dfrac{\sin x}{\cos x}$ |

$\therefore LS = RS$

$\therefore$ QED

b) $\dfrac{1 + \cot x}{\csc x} = \sin x + \cos x$

$LS = \dfrac{1 + \cot x}{\csc x}$

$RS = \sin x + \cos x$

$= \dfrac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$= \left(1 + \dfrac{\cos x}{\sin x}\right) \times \dfrac{\sin x}{1}$ $\longrightarrow = \sin x + \dfrac{\cos x}{\cancel{\sin x}}(\cancel{\sin x})$

$= \left(\dfrac{\sin x + \cos x}{\cancel{\sin x}}\right) \times \dfrac{\cancel{\sin x}}{1}$

$= \sin x + \cos x$

c) $\dfrac{\cos\theta - 1}{1 - \sec\theta} = \dfrac{\cos\theta + 1}{1 + \sec\theta}$

$LS = \dfrac{(\cos\theta - 1)}{1 - \sec\theta} \times \dfrac{(1 + \sec\theta)}{1 + \sec\theta}$

$= \dfrac{\cos\theta + \boxed{\cos\theta\sec\theta} - 1 - \sec\theta}{1 - \sec^2\theta}$

$= \dfrac{\cos\theta + 1 - 1 - \sec\theta}{1 - \sec^2\theta}$

$= \dfrac{\cos\theta - \sec\theta}{1 - \sec^2\theta}$

$\cos\theta \cdot \sec\theta$
$= \cos\theta \cdot \dfrac{1}{\cos\theta}$
$= 1$

change into $\dfrac{1}{\cos\theta}$

$RS = \dfrac{\cos\theta + 1}{1 + \sec\theta} \times \dfrac{1 - \sec\theta}{1 - \sec\theta}$

$= \dfrac{\cos\theta - \boxed{\cos\theta\sec\theta} + 1 - \sec\theta}{1 - \sec^2\theta}$

$= \dfrac{\cos\theta - \sec\theta}{1 - \sec^2\theta}$

$\cos\theta(\sec\theta)$
$= \cos\theta \cdot \dfrac{1}{\cos\theta}$

d) $\tan\alpha \sin\alpha + \cos\alpha = \sec\alpha$

$LS = \tan\alpha \sin\alpha + \cos\alpha$

$= \frac{\sin\alpha}{\cos\alpha} \cdot \sin\alpha + \cos\alpha$

$= \overset{1}{\boxed{\frac{\sin^2\alpha}{\cos\alpha} + \frac{\cos^2\alpha}{\cos\alpha}}}$

$= \frac{1}{\cos\alpha}$

$RS = \sec\alpha$

$= \frac{1}{\cos\alpha}$

$\therefore LS = RS$

$\therefore$ QED

$x^4 - y^4$
$= (x^2 - y^2)(x^2 + y^2)$

e) $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

$LS = \sin^4 x - \cos^4 x$ $\boxed{\text{dos}}$

$= (\sin^2 x - \cos^2 x)(\overset{1}{\sin^2 x + \cos^2 x})$

$= \sin^2 x - \cos^2 x$

$RS = \sin^2 x - \cos^2 x$

$\therefore LS = RS$

$\therefore$ QED

f) $\sin x - \sin x \cos^2 x = \sin^3 x$

f) $\sin x - \sin x\cos^2 x = \sin^3 x$

$LS = \sin x(1-\cos^2 x)$

$= \sin x(\sin^2 x)$

$= \sin^3 x$

$RS = \sin^3 x$

$\because LS = RS$

$\therefore$ QED

$(\sin x - \cos x)(\sin x - \cos x)$

g) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

$= \sin^2 x \boxed{-2\sin x\cos x} + \cos^2 x + \sin^2 x \boxed{+2\sin x\cos x} + \cos^2 x$

$= 1 + 1$

$= 2$

$RS = 2$

$\because LS = RS$

$\therefore$ QED

h) $\dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \dfrac{3+\sin x}{1-\sin x}$

Strategy 1

$LS = \dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x}$

$RS = \dfrac{3+\sin x}{1-\sin x} \times \dfrac{1+\sin x}{1+\sin x}$

$= \dfrac{3 + 3\sin x + \sin x + \sin^2 x}{1-\sin^2 x}$

$= \dfrac{3 + 4\sin x + \sin^2 x}{\cos^2 x}$

Strategy 2

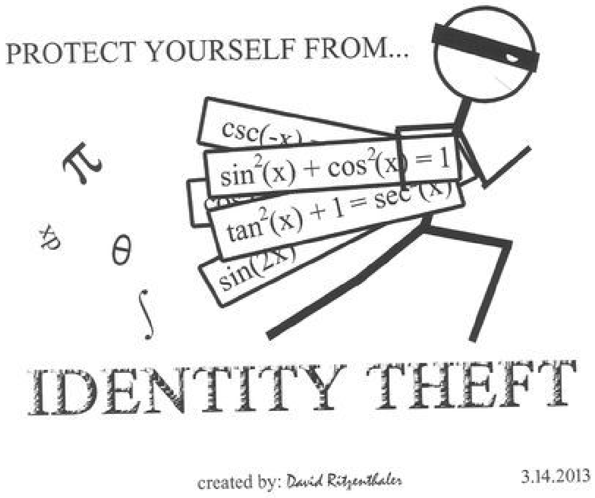
$= \dfrac{(\sin x + 3)(\sin x + 1)}{1-\sin^2 x}$

$= \dfrac{(\sin x + 3)\cancel{(\sin x + 1)}}{(1-\sin x)\cancel{(1+\sin x)}}$

$= \dfrac{\sin x + 3}{1-\sin x}$

p. 273 #7, 8, 9

Handout #3hln, 4aeghijmnor




p. 273 #7, 8, 9

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