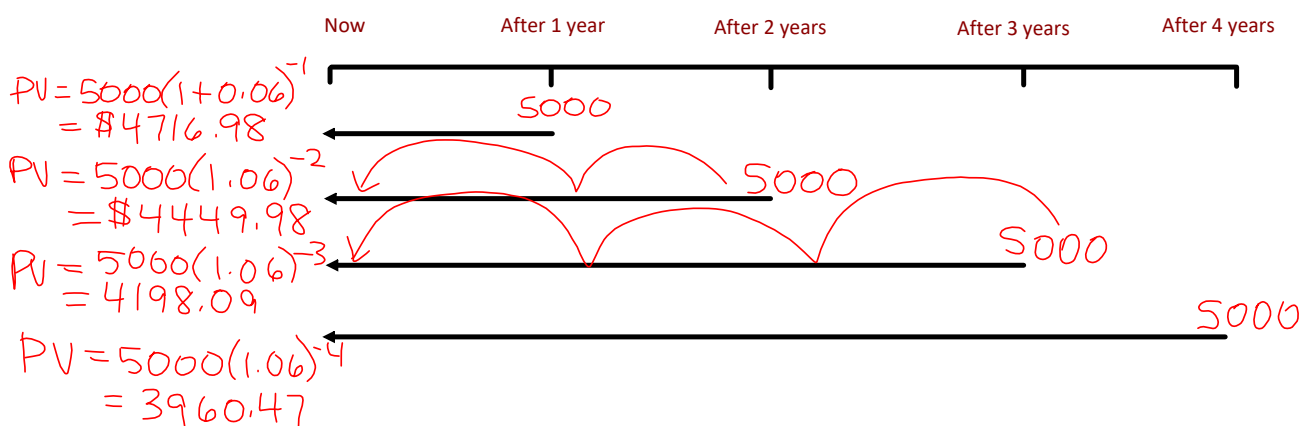


7.4 Present Value of an Annuity

Present Value of an Annuity: The amount of money (principal) that must be invested/borrowed NOW to provide a given series of equal payments at equal intervals of time.

Recall: $PV = A(1+i)^{-n}$

Ex. 1 Next year, Jane is going back to university for a Ph.D. in psychology. She wants to know how much money to deposit now into an account that pays 6%/a, compounded annually, to provide a \$5000 payment each year for 4 years, with the first payment due a year from now.



- What is the value of each deposit at the time of the deposit?

$$4716.98 + 4449.98 + 4198.09 + 3960.47 = \$17325.52$$

Present Value of an Annuity Formula:

$$PV = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$R = \frac{PV i}{1 - (1+i)^{-n}}$$

- Use this to find the present value.
- Use this to find the regular payment.

Don't worry we are going to give you all these formulas :)



where

PV = Present Value

R = Regular payment (made at the end of the compounding period)

i = interest rate per compound pd.

n = # of compound periods/# of payments (must be equal to use formula)

Ex. 1 James wants to invest now so that he will receive \$700 every month for 5 years. How much should he invest now at 4.3%/a compounded monthly to achieve this? $PV = ?$

By Hand

$$PV = ?$$

$$R = 700$$

$$i = \frac{0.043}{12}$$

$$n = 5 \times 12 = 60$$

$$PV = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$= 700 \left[\frac{1 - \left(1 + \frac{0.043}{12}\right)^{-60}}{\left(\frac{0.043}{12}\right)} \right]$$

By TVM

$$N = 60$$

$$I\% = 4.3$$

$$PV = ?$$

$$PMT = 700$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END BEGIN

$$PV = \$37\,731.35$$

Interest?

James gets

$$\$700 \times 60$$

$$= 42\,000$$

Interest

$$= 42\,000 - 37\,731.35$$

$$= \$4\,268.65$$

Ex. 2 Charlie has won the lottery prize of a lump sum payment of \$78 000.
 He has placed the money into an account at 6.3%/a compounded semi-
 annually and plans to withdraw an equal payment every 6 months for 10
 years. How big will the payment be?

By Hand

$$PV = 78000$$

$$R = ?$$

$$i = \frac{0.063}{2}$$

$$n = 10 \times 2$$

$$= 20$$

$$R = \frac{PV \cdot i}{\left[1 - (1+i)^{-n}\right]}$$

$$= \frac{78000 \left(\frac{0.063}{2}\right)}{\left[1 - \left(1 + \frac{0.063}{2}\right)^{-20}\right]}$$

By TVM

$$N = 20$$

$$I\% = 6.3$$

$$PV = 78000$$

$$PMT = ?$$

$$FV = 0$$

$$P/Y = 2$$

$$C/Y = 2$$

$$PMT: \text{END BEGIN}$$

$$PMT: \$5315.80$$

Ex 3. You win Cash For Life!!!!

\$1000 a week (for 25 years) $\rightarrow \$1000 \times 25 \times 52$
 $= 1\,300\,000$

How much does the lottery need to pay us off???

$$PV = ?$$

$$R = 1000$$

$$I = 5$$

$$N = 25 \times 52$$
$$= 1300$$

$$P_1 = 52$$

$$PCY = 52$$

$$PV = \$741\,856.01$$



Homework
Pg. 461
#C2,3bc,
4,6-9,11,12,14
(Graphing Calculator
4,6,9)