16 a) 2, 6, 12, 20, 30

4 6 8 10

2(2) 2(3) 2(4)

$t_n = t_{n-1} + 2n$

1 2 3 4

b) −1, 0, 3, 12 $\quad t_n = t_{n-1} + 3^{n-2}$

+1 +3 +9

3^0

2 3 4

13 a) −4, 8, −16, 32 $\quad -4(-2)^{n-1}$

×−2 ×−2 ×−2

① ② ③

b) $\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}$ $\quad \frac{n}{2n-1}$

6.2 - Arithmetic Sequences

A sequence where there is a common difference, d, between consecutive terms. The same value is added or subtracted to a term to generate the next term.

$d = t_2 - t_1$

eg. 3, 5, 7, 9, 11, ... d = 2

5, 1, -3, -7, ... d = −4

0, 5, 10, 15, 20,.. d = 5

$5 - 3 = 2$

Notice the pattern:

1, 4, 7, 10, 13

1, 1 + 1(3), 1 + 2(3), 1 + 3(3), 1+4(3) a = 1 d = 3

a, a + 1d, a + 2d, a + 3d, a + 4d

always 1 less than term #

Arithmetic Sequence Formula

$t_n = a + (n - 1)d$

where a is the first term and d is the common difference

This means find the general formula which works to find any term in the sequence.
Must be simplified.

Ex. 1 Determine t_n for each.

a) 7, 3, -1, -5, ...

-4 -4

$a = 7$
$d = -4$

$t_n = a + (n-1)d$
$= 7 + (n-1)(-4)$
$= 7 - 4n + 4$
$t_n = 11 - 4n$

b) -5, -3, -1, 1, ...

$a = -5$
$d = 2$

$t_n = a + (n-1)d$
$= -5 + (n-1)(2)$
$= -5 + 2n - 2$
$t_n = 2n - 7$

Ex. 2 Determine the # of terms in each sequence.

a) 2, 5, 8, ..., 155

$n = ?$

$a = 2$
$d = 3$
$t_n = 155$

$t_n = a + (n-1)d$
$155 = 2 + (n-1)(3)$
$\frac{153}{3} = \frac{(n-1)(3)}{3}$
$51 = n - 1$
$52 = n$

b) 1, -1, -3, ..., -199

$a = 1$
$d = -2$
$t_n = -199$

$t_n = a + (n-1)d$
$-199 = 1 + (n-1)(-2)$
$\frac{-200}{-2} = \frac{(n-1)(-2)}{-2}$
$100 = n - 1$
$101 = n$

Ex. 3 Insert two numbers between 17 and 59, so that the four numbers form an arithmetic sequence.

17, ___, ___, 59

d d d

$59 - 17 = 42$
$\frac{42}{3} = 14$

$a = 17$
$n = 4$
$t_n = 59$
$d = ?$

$t_n = a + (n-1)d$
$59 = 17 + (3)(d)$
$d = 14$

4. Determine, a, d, and t_n for each arithmetic sequence.

Solve algebraically

a) $t_4 = 13$, $t_{17} = 39$

$t_n = a + (n-1)d$

$13 = a + (4-1)d$

$13 = a + 3d$ (1)

$39 = a + (17-1)d$

$39 = a + 16d$ (2)

$13 = a + 3d$ (1)

(2) - (1) $\quad \frac{26}{13} = \frac{13d}{13}$

$2 = d$

Sub $d = 2$ into (1)

$13 = a + 3(2)$

$13 - 6 = a$

$7 = a$

$t_n = a + (n-1)d$

$t_n = 7 + (n-1)(2)$

$t_n = 7 + 2n - 2$

$t_n = 5 + 2n$

b) $t_{10} = -67$, $t_{43} = -298$

$a = -4$

$d = -7$

$t_n = -7n + 3$

p. 385 #3abfh, 4bc, 6, 7, 9ac, 10ac, 11ac, 13, 15, 20, 21

p. 385 #3abfh, 4bc, 6, 7, 9ac, 10ac, 11ac, 13, 15, 20, 21