

$$16a) 2, 6, 12, 20, 30$$

$\checkmark \checkmark \checkmark \checkmark$

$4, 6, 8, 10$

$2(2), 2(3), 2(4)$

$$t_n = t_{n-1} + 2n$$

b)

-1	0	3	12
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$t_n = t_{n-1} + 3^{n-2}$

Diagram showing the sequence: -1, 0, 3, 12. Arrows indicate additions: +1 from -1 to 0, +3 from 0 to 3, +9 from 3 to 12. A red circle is around the number 2 above the sequence.

13 a) $\overline{-4, 8, -16, 32}$

$t_n = -4(-2)^{n-1}$

Diagram showing the sequence: -4, 8, -16, 32. Arrows indicate multiplications by -2: $\times -2$ from -4 to 8, $\times -2$ from 8 to -16, $\times -2$ from -16 to 32. Circles ①, ②, ③ are below the sequence.

b) $\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}$

$\frac{n}{2n-1}$

6.2 - Arithmetic Sequences

A sequence where there is a common difference, d , between consecutive terms. The same value is added or subtracted to a term to generate the next term.

$$d = t_2 - t_1$$

eg. $3, 5, 7, 9, 11, \dots$
 $+2 +2 +2 +2$
 $5, 1, -3, -7, \dots$
 $0, 5, 10, 15, 20, \dots$

$$d = 2$$
$$d = -4$$
$$d = 5$$

$$5 - 3 = 2$$

Notice the pattern:

$$\begin{array}{cccccc} 1, & 4, & 7, & 10, & 13 \\ \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} \end{array}$$

$+3$ $+3$ $+3$ $+3$

$$1, 1 + 1(3), 1 + 2(3), 1 + 3(3), 1 + 4(3)$$

$$a = 1 \quad d = 3$$

$$a, \quad a + 1d, \quad a + 2d, \quad a + 3d, \quad a + 4d$$

always 1 less than term #

Arithmetic Sequence Formula

$$t_n = a + (n - 1)d$$

where a is the first term and d is the common difference

Ex. 1 Determine t_n for each.

This means find the general formula which works to find any term in the sequence.
Must be simplified.

a) $7, 3, -1, -5, \dots$

$$\begin{aligned} a &= 7 & t_n &= a + (n-1)d \\ d &= -4 & &= 7 + (n-1)(-4) \\ & & &= 7 - 4n + 4 \\ & & \boxed{t_n = 11 - 4n} & \end{aligned}$$

b) $-5, -3, -1, 1, \dots$

$$\begin{aligned} a &= -5 & t_n &= a + (n-1)d \\ d &= 2 & &= -5 + (n-1)(2) \\ & & &= -5 + 2n - 2 \\ & & \boxed{t_n = 2n - 7} & \end{aligned}$$

Ex. 2 Determine the # of terms in each sequence.

a) $2, 5, 8, \dots, 155$

$$\begin{aligned} a &= 2 & t_n &= 155 \\ d &= 3 & & \\ t_n &= a + (n-1)d & & \\ 155 &= 2 + (n-1)(3) & & \\ \frac{153}{3} &= \frac{(n-1)(3)}{3} & & \\ 51 &= n-1 & & \\ 52 &= n & & \end{aligned}$$

b) $1, -1, -3, \dots, -199$

$$\begin{aligned} a &= 1 & t_n &= a + (n-1)d \\ d &= -2 & -199 &= 1 + (n-1)(-2) \\ t_n &= -199 & -200 &= \frac{(n-1)(-2)}{-2} \\ & & 100 &= n-1 \\ & & 101 &= n \end{aligned}$$

Ex. 3 Insert two numbers between 17 and 59, so that the four numbers form an arithmetic sequence.

$$17, \frac{31}{d}, \frac{45}{d}, 59$$

$$\begin{aligned} \frac{59-17}{3} &= 42 \\ &= 14 \end{aligned}$$

$$\begin{aligned} a &= 17 \\ n &= 4 \\ t_n &= 59 \\ d &=? \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ 59 &= 17 + (3)(d) \\ d &= 14 \end{aligned}$$

4. Determine, a , d , and t_n for each arithmetic sequence.

Solve algebraically

a) $t_4 = 13$, $t_{17} = 39$

$$\begin{aligned} t_n &= a + (n-1)d \\ 13 &= a + (4-1)d \\ 13 &= a + 3d \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} 39 &= a + (17-1)d \\ 39 &= a + 16d \quad \textcircled{2} \\ 13 &= a + 3d \quad \textcircled{1} \\ \hline \textcircled{2} - \textcircled{1} \quad \frac{26}{13} &= \frac{13d}{13} \\ 2 &= d \end{aligned}$$

$$\begin{aligned} \text{Sub } d = 2 \text{ into } \textcircled{1} \\ 13 &= a + 3(2) \\ 13 - 6 &= a \\ 7 &= a \end{aligned}$$

$$\begin{cases} t_n = a + (n-1)d \\ t_n = 7 + (n-1)(2) \\ t_n = 7 + 2n - 2 \\ \boxed{t_n = 5 + 2n} \end{cases}$$

b) $t_{10} = -67$, $t_{43} = -298$

$$\begin{aligned} a &= -4 \\ d &= -7 \end{aligned}$$

$$\boxed{t_n = -7n + 3}$$

**p. 385 #3abfh, 4bc, 6, 7, 9ac
10ac, 11ac, 13, 15, 20, 21**

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