Homework Qs??

16 $4y+1,\ y+4,\ 10-y$

$r = \dfrac{y+4}{4y+1}$ $\qquad r = \dfrac{10-y}{y+4}$

$\dfrac{y+4}{4y+1} = \dfrac{10-y}{y+4}$

$(y+4)(y+4) = (10-y)(4y+1)$

$y^2+8y+16 = 40y+10-4y^2-y$

$5y^2-31y+6=0$

Solve

$(5y-1)(y-6)=0$

$y=\frac{1}{5}$ $\qquad y=6$

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20 $-8,\ x,\ y,\ 72$

$d = x-(-8)$

$d = x+8$

OR

$d = y-x$

$x+8 = y-x$

$\boxed{2x+8 = y}$

$r = \dfrac{y}{x}$ $\qquad r = \dfrac{72}{y}$

$\dfrac{y}{x} = \dfrac{72}{y}$

$y^2 = 72x$

Sub into y $\quad (2x+8)^2 = 72x$

5d $3,\ 3\sqrt{3},\ 9,\ \ldots\ 177\ 147$

$a=3$

$r = \dfrac{3\sqrt{3}}{3}$

$r=\sqrt{3}$

$t_n = ar^{n-1}$

$\dfrac{177\,147}{3} = \dfrac{3(\sqrt{3})^{n-1}}{3}$

$59\,049 = (\sqrt{3})^{n-1}$

$(\sqrt{3})^{20} = (\sqrt{3})^{n-1}$

$(\sqrt{3})^{?}$

6.4 - Arithmetic Series

Arithmetic Sequence: 2, 5, 8, 11, 14, ...

Arithmetic Series: 2 + 5 + 8 + 11 + 14 + ...

An arithmetic series is the SUM of the terms in an arithmetic sequence.
S_n represents the sum of the first n terms

eg. For 2 + 5 + 8 + 11 + ...

$S_2 = 2 + 5$
$S_2 = 7$

$S_3 = 15$

Development of the Arithmetic Series Formula:

Johann Carl Friedrich Gauss
German mathematician

Gauss added the numbers from 1 to 100 by:

$$\begin{array}{r} 1 + 2 + 3 + \dots + 99 + 100 \\ 100 + 99 + 98 + \dots + 2 + 1 \\ \hline 101 \quad 101 \quad 101 \qquad\qquad \end{array}$$

$$\frac{100(101)}{2} \qquad \frac{200(201)}{2}$$

Each sum would = 101
There would be 100 sums all together... but this is two of the series added together so the sum is twice what it should be.

$$S_{100} = \frac{100(101)}{2}$$
$$= 5050$$

Try it out! Nice party trick :)

In general, an arithmetic sequence is: a, a + d, a + 2d, + … + , t_n - d, t_n

3rd last 2nd last last

So, in general, the sum of an arithmetic sequence is: $t_n - 2d$

a + (a + d) + (a + 2d) + … + (t_n - d) + t_n

t_n + t_n - d + $t_n - 2d \ldots + a + d$ + a

a + t_n a + t_n a + t_n a + t_n a + t_n

$$\boxed{t_n = a + (n-1)d}$$

$$S_n = \frac{n(a + a + (n-1)d)}{2}$$

$$S_n = \frac{n(2a + (n-1)d)}{2}$$

$$S_n = \frac{n(a + t_n)}{2}$$

need: # of terms

- first #
- last #

what if we dont know it?

Arithmetic Series Formulas

Any term in an arithmetic sequence/series , t_n can be found using:

$t_n = a + (n - 1)d$

Any sum in an arithmetic series, S_n can be found using:

$$\boxed{S_n = \frac{n(a + t_n)}{2}} \quad \text{or} \quad \boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$$

Know
→ First term
→ Last term

Know
→ first term
→ Common difference

Ex. 1 Find the indicated sum for each series.

a) $4 + 6 + 8 + 10 + \dots \quad S_{42}$

$a = 4$

$d = 2$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{42} = \frac{42}{2}[2(4) + (42-1)(2)]$$

$$S_{42} = 1890$$

b) $5 - 3 - 11 - 19 - \dots, S_{17}$

$$S_{17} = \frac{17}{2}[2(5) + (17-1)(-8)]$$

$$S_{17} = -1003$$

Ex. 2 Find the sum of the series.

a) $2 \overset{d}{+} 5 + 8 + 11 + \dots + 254$

$a = 2$

$t_n = 254$

$n = ?$

$$t_n = a + (n-1)d$$

$$254 = 2 + (n-1)(3)$$

$$\frac{252}{3} = \frac{(n-1)(3)}{3}$$

$$84 = n - 1$$

$$85 = n$$

$$S_n = \frac{n[a + t_n)}{2}$$

$$S_{85} = \frac{85(2 + 254)}{2}$$

$$\boxed{S_{85} = 10\,880}$$

b) $5 + 3 + 1 - 1 - \dots - 401$

$n = 204$

$$\boxed{S_{204} = -40392}$$

Ex. 3 Find the sum of the first 42 terms of an arithmetic series with $t_1 = 7$ and $t_{42} = 212$.

$a = 7$

$t_n = 212$

$n = 42$

$S_n = \frac{n}{2}(a + t_n)$

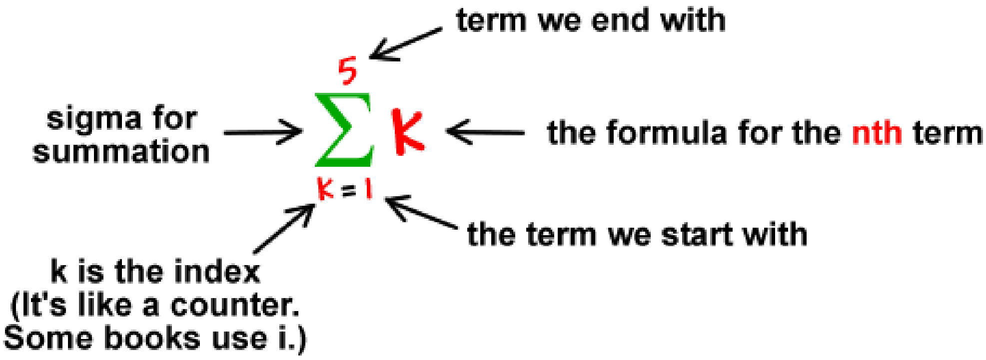
$S_{42} = \frac{42}{2}(7 + 212)$

$S_{42} = 4599$

Homework

8 as many as you need

p. 399 #C2, 1~~a~~df, 3~~b~~c, 4b~~c~~, 6-11, ~~13~~, 14, 18, 20



Unit 6 Quiz #1 (6.1 – 6.3)

1. Determine a formula for the nth term of the sequence $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$
2. Determine the general term for the sequence $-3, -8, -13, -18, \ldots$
3. Algebraically determine the number of terms in the sequence

   9, -18, 36, -72, …, -1152.

4. Algebraically determine t_n for the geometric sequence having $t_5 = 162$ and $t_{10} = 39\ 366$.

$t_n = a + (n-1)d$ $\qquad$ $t_n = ar^{n-1}$