14a)

$$2x,\ 3x+1,\ x^2+2$$

$$d = 3x+1-2x$$
$$= x+1$$

$$d = x^2+2-(3x+1)$$
$$= x^2-3x+1$$

$$x+1 = x^2-3x+1$$
$$0 = x^2-4x$$
$$0 = x(x-4)$$
$$x=0 \qquad x=4$$

b) $x=0$

S_{10}

$a = 2x$

$d = x+1$

$$S_{10} = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$= \frac{10}{2}\left[2(2x)+(10-1)(x+1)\right]$$
$$= 5(4x+9(x+1)]$$
$$= 5(13x+9)$$
$$S_{10} = 65x+45$$

$x=0$

$$S_{10} = 45$$

$x=4$

$$S_{10} = 65(4)+45$$
$$= 305$$

18) $S_9 = 162$ $\qquad$ $S_{12} = 288$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$162 = \frac{9}{2}\left[2a+8d\right)$$

$$288 = \frac{12}{2}\left[2a+11d\right]$$

20) $1+2+4+5+7+8+10+11+\ldots+2999$

999 terms $\longrightarrow$ $3+6+9+12+\ldots \boxed{2997}$

S_{2999}

$a=1$

$d=$

$$S_n = \frac{n}{2}(a+t_n)$$
$$S_{2999} = \frac{2999}{2}(1+2999)$$
$$\boxed{= 4\ 498500}$$

$$S_n = \frac{n}{2}(a+t_n)$$
$$S_{999} = \frac{999}{2}(3+2997)$$
$$= \boxed{1\ 498\ 500}$$

Subtract

$$t_n = 2997$$
$$t_n = a+(n-1)d$$
$$2997 = 3+(n-1)(3)$$
$$\frac{2994}{3} = \frac{(n-1)(3)}{3}$$
$$998 = n-1$$
$$999 = n$$

6.5 - Geometric Series

4, 8, 16, 32, ... geometric sequence

4 + 8 + 16 + 32 + ... geometric series: The **sum** of the terms of a geometric sequence.

Derivation of the Geometric Series Formula

$r^1(ar^{n-1})$

ar^{n-1+1}

ar^n

$$\begin{array}{rl} S_n = & a + \cancel{ar} + \cancel{ar^2} + \cancel{\ldots} + \cancel{ar^{n-1}} \\ - \quad rS_n = & \quad\ \cancel{ar} + \cancel{ar^2} + \cancel{\ldots} + \cancel{ar^{n-1}} + ar^n \\ \hline \end{array}$$

$$S_n - rS_n = a - ar^n$$

$$\frac{S_n(1-r)}{(1-r)} = \frac{a(1-r^n)}{(1-r)}$$

$$\boxed{S_n = \frac{a(1-r^n)}{(1-r)}}$$

$$= \frac{a[\cancel{-1}(r^n - 1)]}{\cancel{-1}(r-1)}$$

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

Geometric Series Formulas

Any term, t_n, can be found using: $t_n = a\,r^{n-1}$

Any sum, S_n, can be found using:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{OR} \quad \frac{a(1 - r^n)}{1 - r}, \text{ where } r \neq 1$$

and:
- a = first term
- r = common ratio
- n = # of terms

Ex. 1 Determine the indicated sum of each series.

a) 4 - 8 + 16 - ... , S_9

$a=4$

$r=-2$

$S_9=?$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_9 = \frac{4[(-2)^9-1]}{-2-1}$$

$$S_9 = 684$$

$$\frac{a(1-r^n)}{r-1}$$

$$= \frac{4(1-(-2)^9)}{1-(-2)}$$

b) 64 + 32 + 16 + ... , S_{12}

$a=64$

$r=\frac{1}{2}$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_{12} = \frac{64[(\frac{1}{2})^{12}-1]}{\frac{1}{2}-1}$$

$$= \frac{64\left(\frac{1}{4096}-\frac{4096}{4096}\right)}{-\frac{1}{2}}$$

$$= \cancel{64}^{1}\left(-\frac{4095}{\cancel{4096}_{64}}\right) \div \left(-\frac{1}{2}\right)$$

$$= -\frac{4095}{\cancel{64}_{32}} \times \left(-\frac{\cancel{2}^{1}}{1}\right)$$

$$S_{12} = \frac{4095}{32}$$

Ex. 2 Determine the sum of the series.

$2 + 6 + 18 + \ldots + 4374$

$a = 2$

$r = 3$

$t_n = 4374$

\# of terms

$t_n = ar^{n-1}$

$$\frac{4374}{2} = \frac{2(3)^{n-1}}{2}$$

$$2187 = 3^{n-1}$$

$$3^7 = 3^{n-1}$$

$$n = 8$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{2[(3)^8 - 1]}{3 - 1}$$

$$S_8 = 6560$$

Ex. 3 What if r = 1? Suppose a = 5 and r =1, find S_{10} for the series.

$5 + 5 + 5 + \ldots + 5$

$S_{10} = 5 \times 10$

$\boxed{S_{10} = 50}$

Homework

p. 407 #C3, 2a~~b~~df,
3bf, 5b~~d~~, 6-12, 16

~~7~~