

## 6.7 - The Binomial Theorem

You have actually used this theorem before...

$$\begin{aligned}(a+b)^2 \\ = (a+b)(a+b) \\ = a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a+b)^3 \\ = (a+b)(a+b)^2 \\ = (a+b)(a^2 + 2ab + b^2) \\ = a^3 + \underline{2a^2b} + ab^2 + \underline{a^2b} \\ + \underline{2ab^2} + b^3 \\ = a^3 + 3a^2b + 3ab^2 + b^3\end{aligned}$$

Investigate the expansion of:

$$\begin{array}{lll}(a+b)^0 & 1 \\ (a+b)^1 & | a + b \\ (a+b)^2 & | a^2 + 2ab + b^2 \\ (a+b)^3 & | a^3 + 3a^2b + 3ab^2 + b^3 \\ (a+b)^4 & | a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\end{array}$$

In general:

$n$ value (exponent)	$(a+b)^n$	Coefficients
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## The Binomial Theorem

The binomial theorem is used to expand  $(a + b)^n$ .

$$(a + b)^n$$

$$= \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n} a^0 b^n$$

$nC_0 \quad nC_1 \quad nC_2 \quad \dots \quad nC_n$

The coefficients are the values of row n in Pascal's Triangle (where n is the exponent to which the binomial is being raised).

Ex. 1 Expand each of the following :

*one more term than exponent  $\rightarrow 7$  terms*

a)  $(a + b)^6$

$$\begin{aligned} &= {}^6 C_0 a^6 b^0 + {}^6 C_1 a^5 b^1 + {}^6 C_2 a^4 b^2 + {}^6 C_3 a^3 b^3 + {}^6 C_4 a^2 b^4 + {}^6 C_5 a^1 b^5 + {}^6 C_6 a^0 b^6 \\ &= a^6 + 6a^5 b + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6ab^5 + b^6 \end{aligned}$$

b)  $(x - 2)^4$

$$\begin{aligned} &= {}^4 C_0 (x)^4 (-2)^0 + {}^4 C_1 (x)^3 (-2)^1 + {}^4 C_2 (x)^2 (-2)^2 + {}^4 C_3 (x)^1 (-2)^3 + {}^4 C_4 (x)^0 (-2)^4 \\ &= x^4 + 4x^3(-2) + 6x^2(4) + 4x(-8) + 16 \\ &= x^4 - 8x^3 + 24x^2 - 32x + 16 \end{aligned}$$

c) The first 3 terms of

$$\begin{aligned}& (3x+2y)^5 \\&= \underbrace{5C_0(3x)^5}_{= 243x^5} + \underbrace{5C_1(3x)^4(2y)}_{= 5(81x^4)(2y)} + 5C_2(3x)^3(2y)^2 \\&= 243x^5 + 810x^4y + 1080x^3y^2\end{aligned}$$

d)  $\left(x^2 + \frac{4}{x}\right)^3$

$$\begin{aligned}& = 3C_0(x^2)^3 + \underbrace{3C_1(x^2)^2\left(\frac{4}{x}\right)^1}_{= x^6} + 3C_2(x^2)^1\left(\frac{4}{x}\right)^2 + 3C_3\left(\frac{4}{x}\right)^3 \\& = x^6 + 3x^4\left(\frac{4}{x}\right) + 3x^2\left(\frac{16}{x^2}\right) + \frac{64}{x^3} \\& = x^6 + 12x^3 + 48 + \frac{64}{x^3}\end{aligned}$$

e) The first four terms of

$$\begin{aligned} & \left(\sqrt{x} - \frac{2}{\sqrt{x^3}}\right)^8 \\ &= 8C_0(\sqrt{x})^8 + 8C_1(\sqrt{x})^7\left(-\frac{2}{\sqrt{x^3}}\right) + 8C_2(\sqrt{x})^6\left(-\frac{2}{\sqrt{x^3}}\right)^2 + 8C_3(\sqrt{x})^5\left(-\frac{2}{\sqrt{x^3}}\right)^3 \\ &= \left(x^{\frac{1}{2}}\right)^8 + 8\left(x^{\frac{7}{2}}\right)\left(-\frac{2}{x^{\frac{3}{2}}}\right) + 28x^6\left(\frac{4}{x^3}\right) + 56x^{\frac{5}{2}}\left(\frac{-8}{x^{\frac{9}{2}}}\right) \\ &= x^4 - 16x^2 + 112 - \frac{448}{x^2} \end{aligned}$$

$$\sqrt{x^3}^5 \\ (x^{\frac{3}{2}})^3$$

$$\begin{aligned} & \frac{5}{2} - \frac{9}{2}x^{-2} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

5 terms  $\Rightarrow n = 4$

Ex. 2 Write  $1 + 12x^3 + 54x^6 + 108x^9 + 81x^{12}$  in the form  $(a+b)^n$

$$\begin{aligned} & nC_0(a)^n \\ &= 1(a)^4 \\ & a^4 = 1 \end{aligned}$$

$$\begin{aligned} & nC_n(b)^n \\ &= 1(b)^4 \end{aligned}$$

$$\begin{aligned} b^4 &= 81x^{12} \\ b &= \sqrt[4]{81x^{12}} \\ &= 3x \end{aligned}$$

# HOMEWORK

**p. 344 #c4, 5acef, 6, 7, 19**

$$\underline{(a+b)^1} = \underline{a} + \underline{b}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$