2.1 Midpoint and Review of y = mx + b

Remember...

$$y = mx + b$$

To write the equation of a line you need slope and y-intercept.

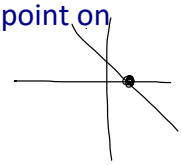
- *Parallel* lines have the same slope.
- *Perpendicular* lines have slopes that are negative reciprocals.

$m = \frac{4}{3}$ $m_{\perp} = -\frac{3}{4}$

- Given two points, find slope using $m = \frac{y_2 - y_1}{x_2 - x_1}$

(x, y)

- *Same x-int* means find the *x*-int by substituting $y = 0$, then use this point, $(x,0)$, as a point on the line to find b.



- Can use any point on the line to substitute along with *m* to find *b*.

Examples: Find the equations of the following lines:

a) passes through C(3,-4) and D(-1,7)

$y = mx + b$ (x_2, y_2)

m (x_1, y_1) b

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-4)}{-1-3} = \frac{11}{-4}$

$m = -\frac{11}{4}$

* use a point

$y = mx + b$

$y = -\frac{11}{4}x + b$

$-4 = -\frac{11}{4}(3) + b$

$-4 = -\frac{33}{4} + b$

$\frac{33}{4} - \frac{16}{4} = b$

$\frac{17}{4} = b$

x, y $(3, -4)$

$y = -\frac{11}{4}x + \frac{17}{4}$

b) perpendicular to $4x + 3y - 7 = 0$ with the same x-intercept as $2x + 3y - 12 = 0$

m b

$4x + 3y - 7 = 0$

$\frac{3y}{3} = -\frac{4x}{3} + \frac{7}{3}$

$y = -\frac{4}{3}x + \frac{7}{3}$

$m = -\frac{4}{3}$

$m_{\perp} = \frac{3}{4}$

* need a point (x-int) $y = 0$

$2x + 3y - 12 = 0$

$2x + 3(0) - 12 = 0$

$2x = 12$

$x = 6$

$\therefore (6,0)$ not b'

$y = mx + b$

$y = \frac{3}{4}x + b$

$0 = \frac{3}{4}(6) + b$

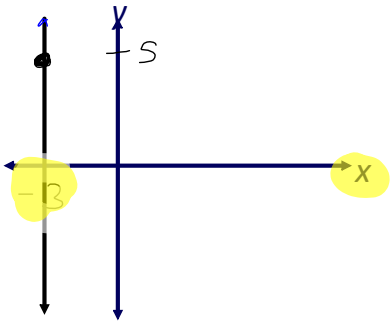
$0 = \frac{9}{2} + b$

$-\frac{9}{2} = b$

$\therefore y = \frac{3}{4}x - \frac{9}{2}$

SPECIAL CASES: Horizontal & Vertical Lines

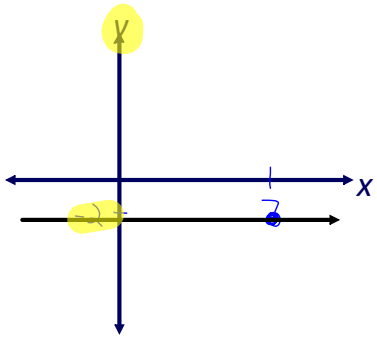
c) a vertical line passing through (-3,5)



the x-coordinate is always -3

$x = -3$

d) a horizontal line passing through (7,-2)



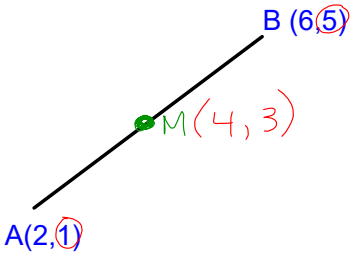
the y-coordinate is always -2

$y = -2$

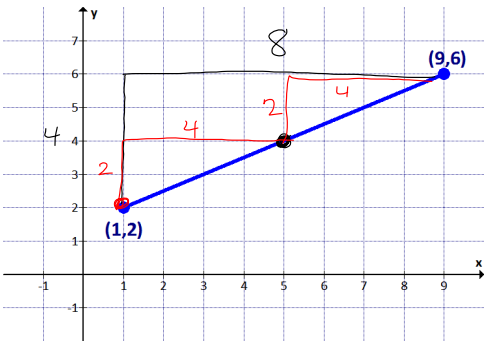
The Midpoint

Notation: $M(x_M, y_M)$

Remember that '**m**' denotes slope!



What are the coordinates of the midpoint of segment AB?



How can you determine the midpoint <u>algebraically</u> given the coordinates of the endpoint?

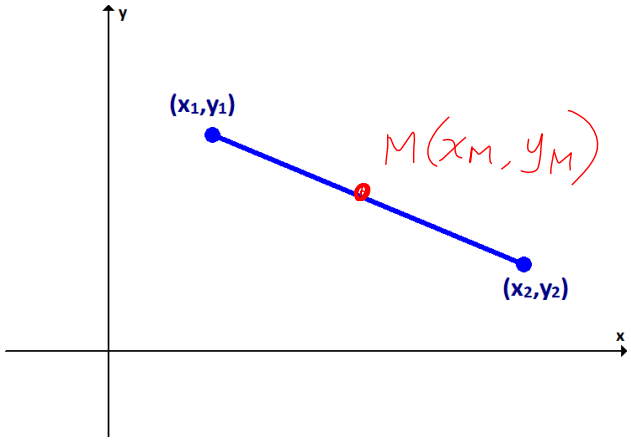
(5,4)

$x_M = \frac{1+9}{2}$ $\quad y_M = \frac{2+6}{2}$

$x_M = 5$ $\quad y_M = 4$

The coordinates of the midpoint of a line segment are:

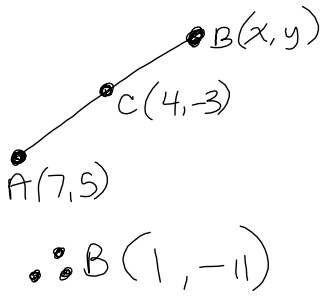
$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



Ex. 1 Find the midpoint of the line segment AB where A(2,-4) and B(-3,5).

$$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{2+(-3)}{2}, \frac{-4+5}{2}\right)$$
$$= \left(-\frac{1}{2}, \frac{1}{2}\right)$$

Ex. 2 C(4, -3) is the midpoint of a line segment with endpoints A(7, 5) and B. Determine the coordinates of B.



$\therefore B(1, -11)$

$$x_M = \frac{x_1+x_2}{2}$$
$$2x(4) = \left(\frac{7+x_B}{2}\right)x2$$
$$8 = 7 + x_B$$
$$8-7 = x_B$$
$$1 = x_B$$

$$y_M = \frac{y_1+y_2}{2}$$
$$-3 = \frac{5+y_B}{2}$$
$$-3(2) = 5 + y_B$$
$$-6-5 = y_B$$
$$y_B = -11$$

Ex.3 The diameter of a circle has endpoints A(4, -3) and B (-3, 5). Find the centre of the circle.

$$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{4+(-3)}{2}, \frac{-3+5}{2}\right)$$
$$= \left(\frac{1}{2}, 1\right)$$