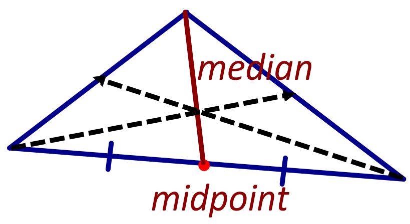
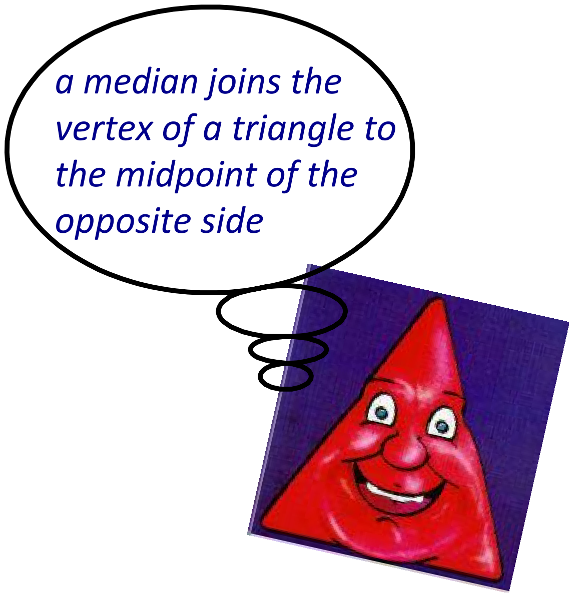
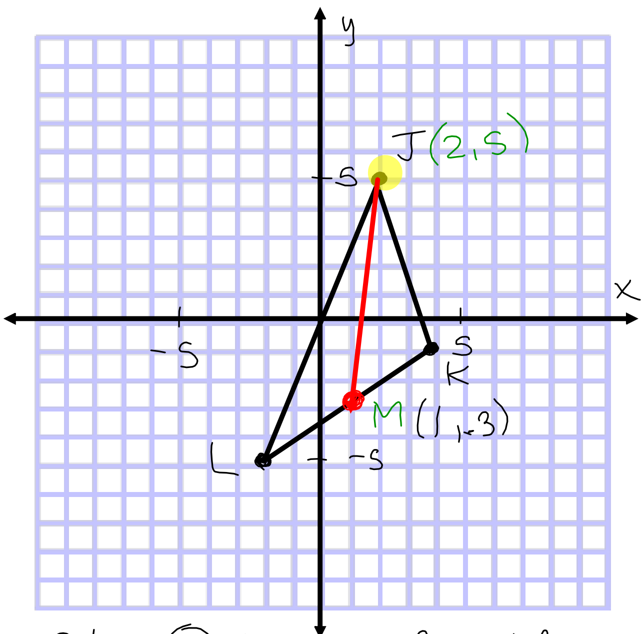
2.2 Equations of Medians, Altitudes and Right Bisectors

A. MEDIANS





Ex. 1: Determine the equation of the median from *J* for the triangle with vertices *J*(2,5), *K*(4,-1) and *L*(-2,-5).



Equation
→ m → need 2 points on median line
→ b

Step ① Midpoint LK

$$M_{LK} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+4}{2}, \frac{-5-1}{2}\right)$$

$$M_{LK} = (1,-3)$$

Step ② Slope of JM

$$m_{JM} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{5-(-3)}{2-1}$$

$$= 8$$

Step ③ → find b

$y = mx + b$ → $m = 8$, $J(2,5)$

$$5 = 8(2) + b$$

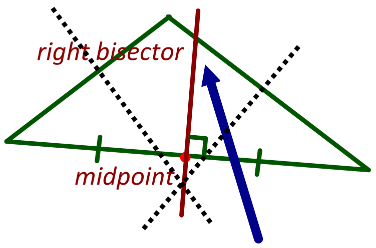
$$5 = 16 + b$$

$$5 - 16 = b$$

$$-11 = b$$

$$\therefore \boxed{y = 8x - 11}$$

B. PERPENDICULAR (OR RIGHT) BISECTORS

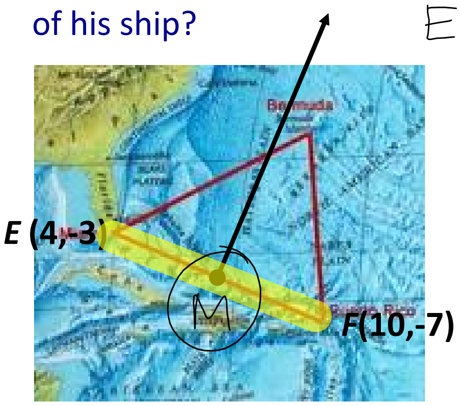


a perpendicular bisector passes through a midpoint at 90⁰

look: does not have to go through vertex

<u>Ex. 2</u> Below is one of the most famous triangles... THE BERMUDA TRIANGLE! A ship plans to take the path of the perpendicular bisector from the segment *EF*. He wishes to be tracked the whole way. Can you determine the equation of his ship?

$E(4,-3)$ $F(10,-7)$



Equation → m (⊥ slope)
→ b (need a point)
→ M of EF

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ } need 2 points.

Step ① Find Slope of EF

$$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7-(-3)}{10-4}$$
$$= \frac{-4}{6}$$
$$= -\frac{2}{3}$$

$m_{\perp} = \frac{3}{2}$

Step ② Midpoint of EF

$$M_{EF} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{4+10}{2}, \frac{-3+(-7)}{2}\right)$$
$$= (7,-5)$$

Step ③ Find 'b'

$m = \frac{3}{2}$, $M(7,-5)$

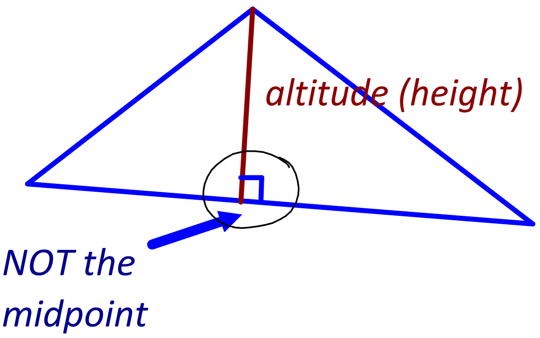
$$y = mx + b$$
$$-5 = \frac{3}{2}(7) + b$$
$$-5 = \frac{21}{2} + b$$
$$-5 - \frac{21}{2} = b$$
$$-\frac{10}{2} - \frac{21}{2} = b$$
$$-\frac{31}{2} = b$$

∴ Perpendicular Bisector

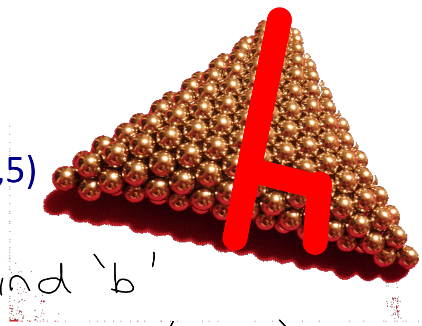
$$y = \frac{3}{2}x - \frac{31}{2}$$

C. ALTITUDES

An altitude joins the vertex of a triangle to its opposite side at 90º



Ex. 3 Determine the equation of the altitude from *A*. A(4,-1)



Equation → m ($\perp$ slope)
→ b (need a point)
A(4,-1)

Step ① Slope CB

$$m_{CB} = \frac{y_2-y_1}{x_2-x_1}$$
$$= \frac{5-3}{-1-(-7)}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

$$m_{\perp} = -3$$

Step ② Find 'b'

$m=-3$ Point $(4,-1)$

$$y = mx+b$$
$$-1 = -3(4)+b$$
$$-1 = -12+b$$
$$-1+12 = b$$
$$11 = b$$

$$\boxed{\therefore y = -3x+11}$$

12, 13, 29

12 $P(a,b) \quad Q(3a, 2b)$

$$M_{PQ} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{a+3a}{2}, \frac{b+2b}{2}\right)$$

$$= \left(\frac{4a}{2}, \frac{3b}{2}\right)$$

$$= \left(2a, \frac{3b}{2}\right)$$

13 $C(6,5)$, $M(4,2)$, D

$$x_M = \frac{x_1+x_2}{2}$$

$$2 \times (4) = \left(\frac{6+x_D}{2}\right) \times 2$$

$$8 = 6 + x_D$$

$$8 - 6 = x_D$$

$$\boxed{2 = x_D}$$

$$y_M = \frac{y_1+y_2}{2}$$

$$2 = \frac{5+y_D}{2}$$

$$2(2) = 5 + y_D$$

$$4 - 5 = y_D$$

$$\boxed{-1 = y_D}$$

$D(2,-1)$

29 $M(d,3)$, $P(3,-4)$, $Q(11,c)$

$$x_M = \frac{x_1+x_2}{2}$$

$$d = \frac{3+11}{2}$$

$$\boxed{d = 7}$$

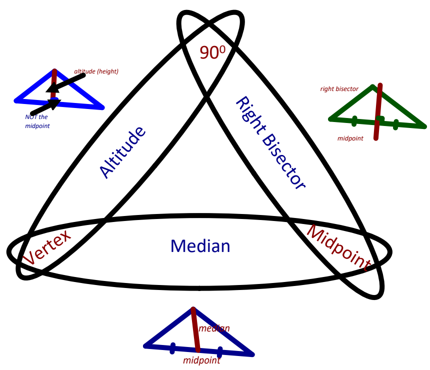
$$y_M = \frac{y_1+y_2}{2}$$

$$3 = \frac{-4+c}{2}$$

$$6 = -4 + c$$

$$6 + 4 = c$$

$$\boxed{10 = c}$$



Homework

Pg. 66 #4b, 8, 17, 23
Pg. 90 #18
Pg. 100 #4