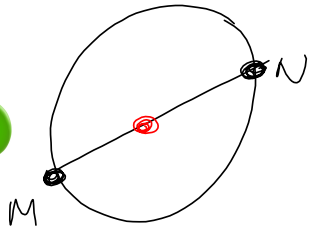
2.5 Problems: Slope, Length and Midpoint

Put it all together now.....

Ex.1 Determine the radius of a circle with endpoints of a diameter M(-3,5) and N(9,7).



$$d_{MN} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(9-(-3))^2 + (7-5)^2}$$
$$= \sqrt{(12)^2 + (2)^2}$$
$$= \sqrt{144+4}$$
$$= \sqrt{148}$$
$$\approx$$

$\therefore$ Radius $= \frac{\sqrt{148}}{2}$

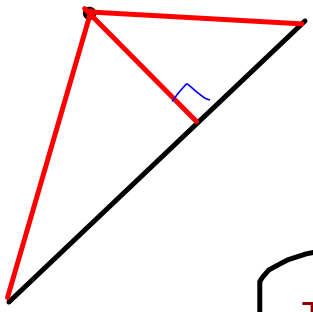
① Midpoint
② Distance Mid → End

OR

① Diameter d_{MN}
② Radius ÷ 2

Investigate!

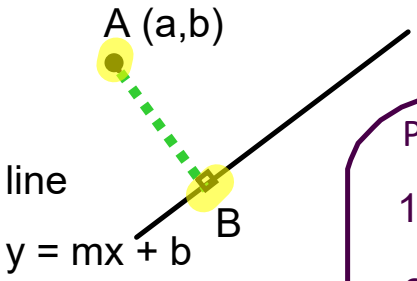
What is the shortest distance from the point to the line?



- Draw a line and a point.
- Connect the point and line with several line segments.
- Measure the line segments.
- Which is the shortest? What are its properties?

The shortest distance from a point to a line is always the length of the segment that is perpendicular to the line.

Outline a PLAN to find the distance from A to B.



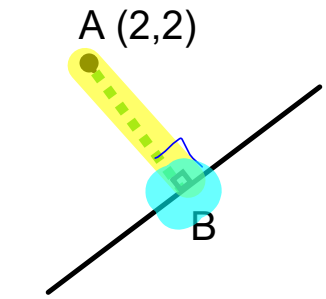
- we know the equation of the line and the coordinates of point A.

Plan:

1. slope of line, then slope of AB $(m_{\perp})$
2. equation of line AB (using slope and point A)
3. elimination/substitution to find intersection of AB and line (coordinates of B)
4. distance formula to find length of AB

recall: $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex. 2 Find the shortest distance from (2,2) to the line $y = x+1$



$m=1$

We need B to get the distance from A to B...
How do we find the coordinates of B?

★ B is the point of intersection of the 2 lines!

find B using substitution or elimination
∴ we need the equation of the line AB

① Equation of AB

$m_\perp = -1$

$y = mx + b$
$2 = -1(2) + b$
$2 + 2 = b$
$4 = b$
∴ $y = -x + 4$

② POI (B)

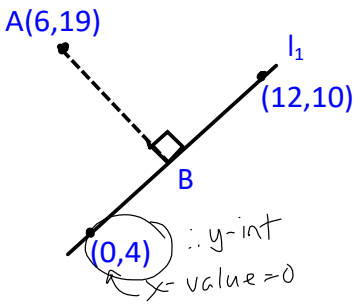
$y = x + 1$ ①
$y = -x + 4$ ②

① + ② $\frac{2y}{2} = \frac{5}{2}$
$y = \frac{5}{2}$
Sub $y = \frac{5}{2}$ into ①
$\frac{5}{2} = x + 1$
$\frac{5}{2} - 1 = x$
$\frac{5}{2} - \frac{2}{2} = x$
$\frac{3}{2} = x$
∴ $B\left(\frac{3}{2}, \frac{5}{2}\right)$

③ Distance (AB)

$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{\left(2-\frac{3}{2}\right)^2 + \left(2-\frac{5}{2}\right)^2}$
$= \sqrt{\left(\frac{4}{2}-\frac{3}{2}\right)^2 + \left(\frac{4}{2}-\frac{5}{2}\right)^2}$
$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$
$= \sqrt{\frac{1}{4} + \frac{1}{4}}$
$= \sqrt{\frac{2}{4}}$
$= \sqrt{\frac{1}{2}}$ Exact
$\doteq 0.71$ Approximate

Ex.3 Given the line containing the point (0,4) and (12,10), determine the SHORTEST distance from A(6,19) to the line.



How is this question different from the last one?

☆ We dont have the equation of Line ①

① Equation of L①

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-4}{12-0} = \frac{6}{12} = \frac{1}{2}$

$y = mx + b$
$4 = \frac{1}{2}(0) + b$
$4 = b$

$\therefore y = \frac{1}{2}x + 4$

② Equation of AB

$m_{\perp} = -2$

$y = mx + b$
$19 = -2(6) + b$
$19 = -12 + b$
$19 + 12 = b$
$31 = b$

$\therefore y = -2x + 31$

⑤ POI

$y = \frac{1}{2}x + 4$ ① $\xrightarrow{\times 2}$ $2y = x + 8$

$y = -2x + 31$ ②

Sub ① into ②

$\frac{1}{2}x + 4 = -2x + 31$
$\frac{1}{2}x + 2x = 31 - 4$
$\frac{1}{2}x + \frac{4}{2}x = 27$
$2x\left(\frac{5}{2}x\right) = 27 \times 2$
$\frac{5x}{5} = \frac{54}{5}$
$x = \frac{54}{5}$

Sub $x = \frac{54}{5}$ into ①

$y = \frac{1}{2}\left(\frac{54}{5}\right) + 4$
$y = \frac{54}{10} + \frac{40}{10}$
$y = \frac{94}{10}$
$y = \frac{47}{5}$

$\therefore B\left(\frac{54}{5}, \frac{47}{5}\right)$

④ distance AB

$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{\left(6 - \frac{54}{5}\right)^2 + \left(19 - \frac{47}{5}\right)^2}$

↓ carry through fractions

d_{AB} = Exact

$d_{AB} \doteq$?

What we are doing is called analytic geometry!

Homework

Pg. 88 #C3,2,3,8,10,24

Handout → do 2 (correct) (not a)