2.8 Verifying Properties of Triangles

↳ algebraically

Type	Features	Need to Show....
equilateral	all sides equal	side lengths are equal → distance
isosceles	two sides are equal	2 side lengths are equal → distance
scalene	no equal sides	3 side lengths are different → distance
right	has a 90° angle	• shows slopes of 2 sides are neg. recip OR • show that 3 sides satisfy Pythagorean relationship

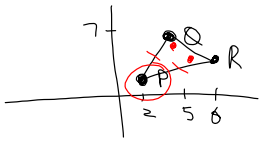
right isosceles?

right scalene?

Example 1: P(2,1), Q(5,7) and R(8,4) are the vertices of the triangle Classify the triangle.

→ find the lengths of all 3 sides.

→ check if 90° (if needed)



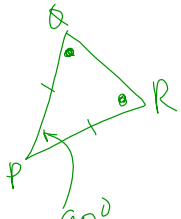
$$L_{PQ} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5-2)^2+(7-1)^2}$$
$$= \sqrt{3^2+6^2}$$
$$= \sqrt{9+36}$$
$$L_{PQ} = \sqrt{45}$$

$$L_{QR} = \sqrt{(8-5)^2+(4-7)^2}$$
$$= \sqrt{3^2+(-3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$

$$L_{PR} = \sqrt{(8-2)^2+(4-1)^2}$$
$$= \sqrt{6^2+3^2}$$
$$= \sqrt{36+9}$$
$$= \sqrt{45}$$

$\because PQ = PR \quad \therefore$ Isosceles

Check if Right angle P.



$$m_{PQ} = \frac{y_2-y_1}{x_2-x_1} = \frac{7-1}{5-2} = \frac{6}{3}$$
$$m_{PQ} = 2$$

$$m_{PR} = \frac{y_2-y_1}{x_2-x_1} = \frac{3}{6}$$
$$m_{PR} = \frac{1}{2}$$

not negative reciprocals

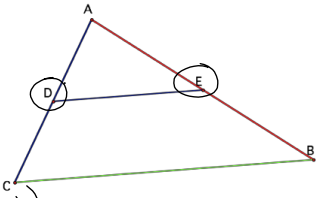
$\therefore$ NOT RIGHT

$\therefore$ Isosceles Triangle

Ex. 2 Classify triangle PQR, given P(-7,1), Q(-8,4) and R(-1,3).

☆ Right.

Ex. 3 The vertices of a triangle are A(-3,4),B(-2,-4), and C(5,-2). *D* is the midpoint of *AC* and *E* is the midpoint of AB. Verify that *DE* is parallel to BC and half its length.



① DE is parallel to BC
find slopes, compare them

Ⓐ Find D and E (Midpoints)

$M_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-3+5}{2}, \frac{4+(-2)}{2}\right)$

$D = (1,1)$

$M_{AB} = \left(\frac{-3-2}{2}, \frac{4+(-4)}{2}\right)$

$E = \left(-\frac{5}{2}, 0\right)$

B(-2,-4)
C(5,-2)

Ⓑ Compare slopes

$m_{DE} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{1-0}{1-(-\frac{5}{2})}$

$= \frac{1}{\frac{2}{2}+\frac{5}{2}}$

$= \frac{1}{\frac{7}{2}}$

$= 1 \div \frac{7}{2}$

$= 1 \times \frac{2}{7}$

$= \frac{2}{7}$

$m_{BC} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-4-(-2)}{-2-5}$

$= \frac{-2}{-7}$

$= \frac{2}{7}$

$\therefore m_{DE} = m_{BC}$

$\therefore DE \parallel BC$

D(1,1)
E$(-\frac{5}{2}, 0)$

Ⓒ Find lengths DE, BC

B(-2,-4)
C(5,-2)

$L_{DE} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{\left(1-(-\frac{5}{2})\right)^2+(1-0)^2}$

$= \sqrt{\left(\frac{2}{2}+\frac{5}{2}\right)^2+1}$

$= \sqrt{\left(\frac{7}{2}\right)^2+1}$

$= \sqrt{\frac{49}{4}+\frac{4}{4}}$

$= \sqrt{\frac{53}{4}}$

$= \frac{\sqrt{53}}{2}$

$\doteq 3.64$

$L_{BC} = \sqrt{(5-(-2))^2+(-2-(-4))^2}$

$= \sqrt{(5+2)^2+(-2+4)^2}$

$= \sqrt{7^2+2^2}$

$= \sqrt{49+4}$

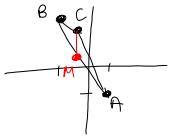
$= \sqrt{53}$

$\doteq 7.28$

$3.64 \times 2 = 7.28$

$\therefore DE = \frac{1}{2} BC$

4. A triangle has vertices A(2,-3), B(-3,5), and C(-2, 4).
Find the centroid. → POI of medians
→ need 2 equations (↳ of the 3 median lines)



$m_{CM_{AB}}$

① Median from C → M_{AB}

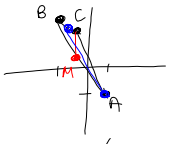
$$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{2+(-3)}{2}, \frac{-3+5}{2}\right)$$
$$= \left(\frac{-1}{2}, \frac{2}{2}\right)$$
$$= \left(-\frac{1}{2}, 1\right)$$

$$m_{CM_{AB}} = \frac{y_2-y_1}{x_2-x_1}$$
$$= \frac{4-1}{-2-\left(-\frac{1}{2}\right)}$$
$$= \frac{3}{-\frac{4}{2}+\frac{1}{2}}$$
$$= \frac{3}{-\frac{3}{2}}$$
$$= 3 \div \left(-\frac{3}{2}\right)$$
$$= 3 \times \left(-\frac{2}{3}\right)$$
$$m = -2$$

Equation
$m = -2$
$b = ?$ $C(-2,4)$

$y = mx + b$
$4 = -2(-2) + b$
$4 = 4 + b$
$4-4 = b$
$0 = b$

∴ Equation of median from C
$\boxed{y = -2x}$



② Equation from A → M_{BC} $\left(-\frac{5}{2}, \frac{9}{2}\right)$
(2,-3)

A(2,-3), B(-3,5), and C(-2, 4).

$$M_{BC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{-3+(-2)}{2}, \frac{5+4}{2}\right)$$
$$= \left(-\frac{5}{2}, \frac{9}{2}\right)$$

$$m_{AM_{BC}} = \frac{y_2-y_1}{x_2-x_1}$$
$$= \frac{\frac{9}{2}-(-3)}{-\frac{5}{2}-2}$$
$$m = -\frac{5}{3}$$

$y = mx + b$ $A(2,-3)$
$y = -\frac{5}{3}x + b$

$-3 = -\frac{5}{3}(2) + b$
$-3 = -\frac{10}{3} + b$
$-\frac{9}{3} + \frac{10}{3} = b$
$\frac{1}{3} = b$

∴ median from A
$y = -\frac{5}{3}x + \frac{1}{3}$

③ POI
$y = -2x$ ①
$y = -\frac{5}{3}x + \frac{1}{3}$ ②

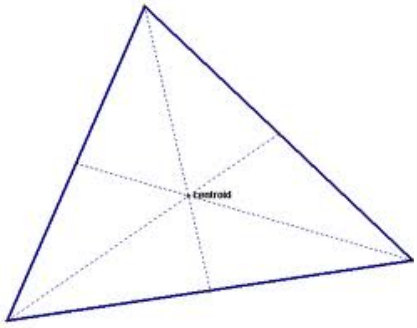
Sub ① into ②
$3(-2x) = -5x + 1$
$-6x = -5x + 1$
$-1 = -5x + 6x$
$-1 = x$

Sub $x = -1$ into ①
$y = -2(-1)$
$y = 2$

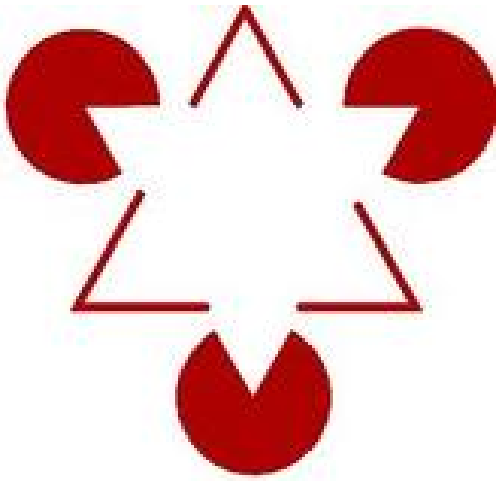
∴ Centroid is at $(-1, 2)$

↓
check in 3rd equation

Ex. 4 (-1,2) is the centroid of triangle with vertices A(2,-3), B(-3,5), and C(-2, 4). Verify that the centroid divides each median in a 2:1 ratio.



Homework
Page 124
#2,4,6,15



Homework Check
- show me 2.7B Extra Practice Handout