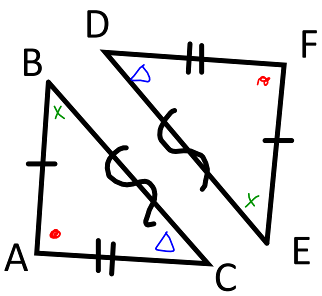
6.1 Properties of Congruent and Similar Triangles

Congruent Triangles

- identical triangles
- all corresponding angles and all corresponding sides are equal
- same shape and size congruent
- the expression $\Delta ABC \cong \Delta FED$ means that ΔABC *is congruent to* ΔFED

The order of the letters shows correspondence: A F, B E, C D

Ex.1 Using the triangles shown:
State the congruent triangles and the 6 known equalities.



$\angle A = \angle F$ $\quad AB = FE$

$\angle B = \angle E$ $\quad AC = FD$

$\angle C = \angle D$ $\quad BC = ED$

* order is important!

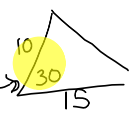
$\Delta ABC \cong \Delta FED$

Conditions for Congruency (3 Theorems)

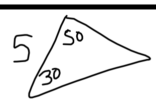
There are three methods to *prove* triangles are congruent:

side side side

1. SSS --> All sides are equal.

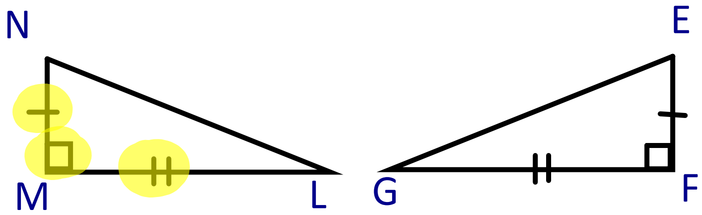


2. SAS --> 2 sides and a CONTAINED angle are equal.
3. ASA --> 2 angles and a side are equal.



Ex.2 Prove that the triangles are congruent and name the authority used.

a)

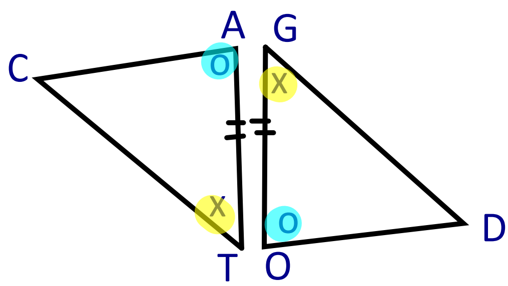


SSS
ASA
SAS

$\angle M = \angle F$
$ML = FG$
$MN = FE$

$\triangle LMN \cong \triangle GFE$, (SAS)

b)



$\angle A = \angle O$
$\angle T = \angle G$
$AT = OG$

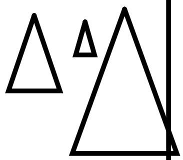
SSS
ASA
SAS

$\triangle CAT \cong \triangle DOG$, (ASA)

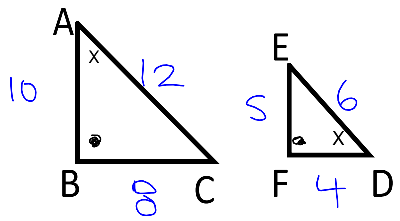
SIMILAR TRIANGLES

- have the same shape BUT a different size
- one triangle is an enlargement/reduction of the other
- the corresponding sides are proportional
- the corresponding angles are equal
- the expression ΔABC~ΔDEF means the triangles are similar (order matters, A-D, B-E, and C-F)

similar



Ex. 3 Given similar ΔABC and ΔDEF, state the similarity, the known equalities (angles) and the known proportions (sides).



$\angle A = \angle D$

$\angle B = \angle F$

$\angle C = \angle E$

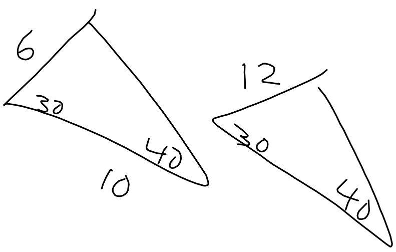
$$\frac{AB}{EF} = \frac{BC}{FE} = \frac{AC}{ED}$$

$\triangle ABC \sim \triangle EFD$

Conditions for Similarity (3 Theorems)

There are 3 methods to *prove* that triangles are similar:

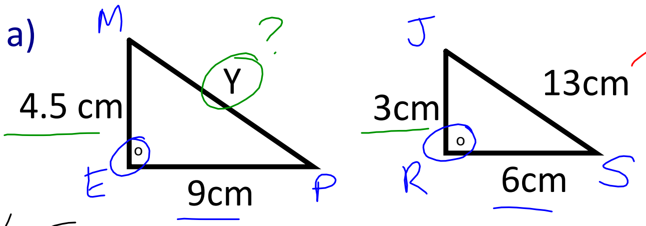
1. SSS --> If 3 pairs of corresponding sides are proportional then the triangles are similar.
2. SAS --> If 2 pairs of corresponding sides are proportional AND the contained angles are equal then the triangles are similar.
3. AA --> If two pairs of corresponding angles are equal then the triangles are similar.



Ex. 4 Prove similarity, then determine the unknowns.

SSS
AA
SAS

a)



$MP = 13 \times 1.5$
$MP = 19.5$
Scale factor

$\angle E = \angle R$

$\frac{ME}{JR} = \frac{EP}{RS}$

$\frac{4.5}{3}$ $\frac{9}{6}$

$= 1.5$ $= 1.5$

Scale factor

$$\boxed{\triangle MEP \sim \triangle JRS}$$

State similarity (SAS) and proof

$$\frac{ME}{JR} = \frac{EP}{RS} = \frac{MP}{JS}$$

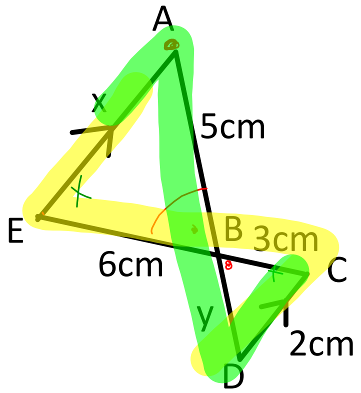
$$\frac{9}{6} = \frac{y}{13}$$

$$13\left(\frac{9}{6}\right) = y$$

$$19.5 = y$$

SSS
AA
SAS

b)



$\angle ABE = \angle DBC$ (OAT)

$\angle A = \angle D$ (PLT-Z)

$\angle E = \angle C$ (PLT-Z)

$$\triangle AEB \sim \triangle DCB \text{ (AA)}$$

$$\frac{EB}{CB} = \frac{AE}{DC} = \frac{AB}{DB}$$

$$\frac{6}{3} = \frac{x}{2} = \frac{5}{y}$$

Scale factor $= 2$

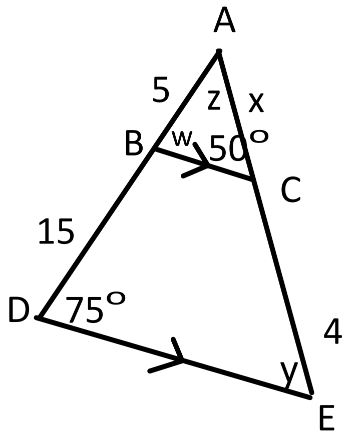
$\frac{6}{3} = \frac{x}{2}$

$x = 4$

$\frac{6}{3} = \frac{5}{y}$

$y = 2.5$

c)



SSS
AA
SAS

p. 333 # C2, C4, 4-9, 14, 15

Quadratics → Forms

- Standard $y = ax^2 + bx + c$
- Vertex $y = a(x-h)^2 + k$
- Factored $y = a(x-r)(x-s)$

Graphing

(→ tables of values)

→ vertex form → vertex, over 1 up 1

over 2 up 4

→ factored form → zeros

~~of~~ → vertex

* 5 points