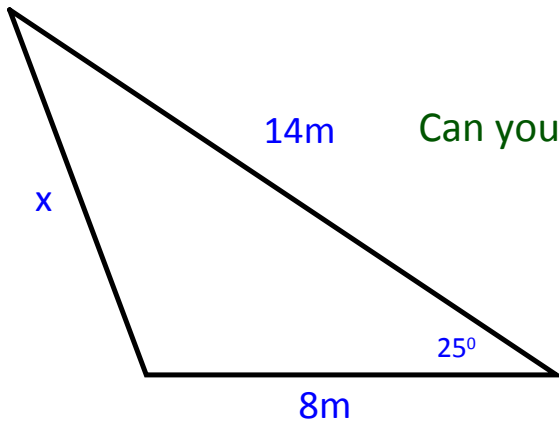


6.6 The Cosine Law



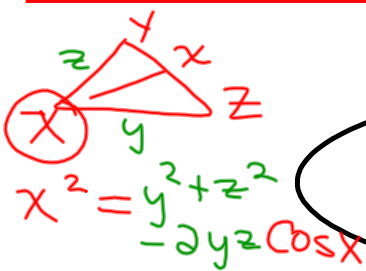
Can you solve for the unknown?

We need a new formula to solve this triangle.

Cosine Law: In $\triangle ABC$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Side length ****use to find a side length****
when given 2 sides and a contained angle



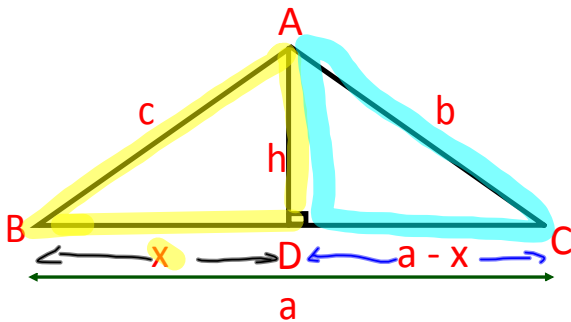
Similar Equations...




$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Can you see the pattern?

Development of the Cosine Law:




- consider $\triangle ABC$ (no 90°) 
- construct an altitude from A 
- notice that there are now 2 right triangles 

In $\triangle BDA$

- Write the Pythagorean theorem

$$c^2 = x^2 + h^2$$

CAH 

- Write the cosine ratio for B

$$\cos B = \frac{x}{c}$$

← isolate 'x'

$$\underline{c \cos B = x}$$


In $\triangle CDA$


- Write the Pythagorean theorem

$$b^2 = (a-x)^2 + h^2$$

$$b^2 = (a-x)(a-x) + h^2$$

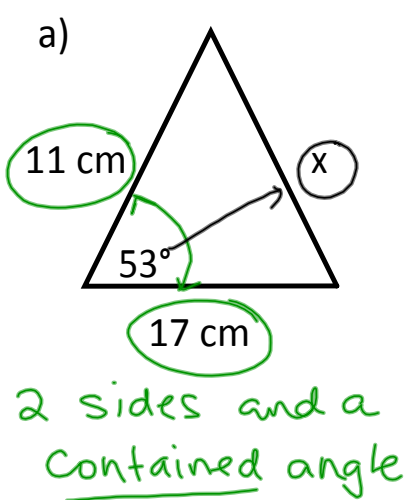
$$b^2 = a^2 - 2ax + x^2 + h^2$$

$$b^2 = a^2 - 2ax + c^2$$

$$b^2 = a^2 + c^2 - 2ax$$


$$b^2 = a^2 + c^2 - 2ac \cos B$$

Ex. 1: Determine the unknown variable using the cosine law.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

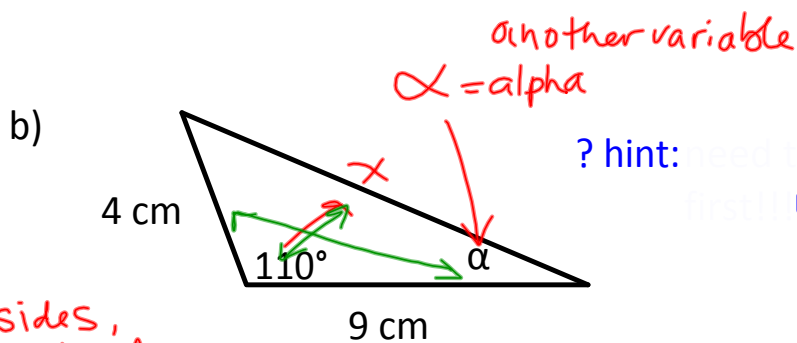
$$x^2 = (11)^2 + (17)^2 - 2(11)(17) \cos 53$$

$$x^2 \doteq 184.9$$

$$x = \pm \sqrt{184.9}$$

$$x \doteq 13.6, \quad x > 0$$

cm



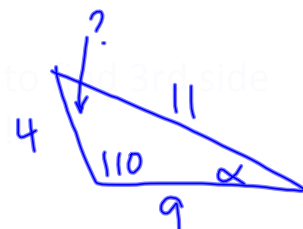
2 sides,
contained
angle

$$\textcircled{1} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 4^2 + 9^2 - 2(4)(9) \cos 110$$

$$x^2 \doteq 122.8$$

$$x \doteq 11 \text{ cm}$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$4^2 = 9^2 + 11^2 - 2(9)(11) \cos A$$

Find A

$$\textcircled{2} \quad \frac{\sin A}{a} = \frac{\sin B}{b}$$

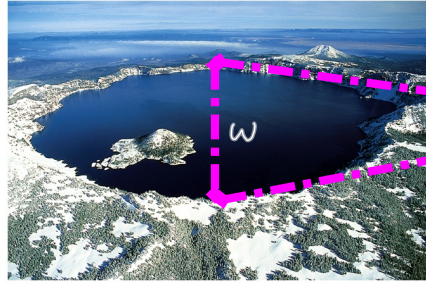
$$\frac{\sin \alpha}{4} = \frac{\sin 110}{11}$$

$$\sin \alpha = \frac{4 \sin 110}{11}$$

$$\alpha = \sin^{-1} \left(\frac{4 \sin 110}{11} \right)$$

$$\alpha \doteq 20^\circ$$

Ex. 2. Find the width of the lake, to the nearest metre, given the following:



2 sides, "inner" angle (contained)

∴ The width of the lake is ~ 6 km

$$a^2 = b^2 + c^2 - 2bc \cos A$$

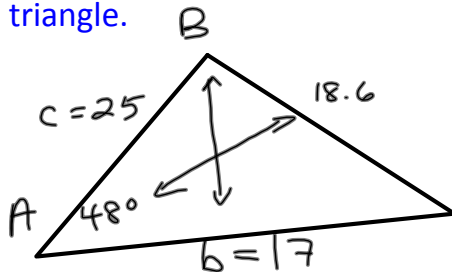
$$w^2 = 10^2 + 12^2 - 2(10)(12) \cos 30$$

$$w^2 = 36.2$$

$$w \approx 6 \text{ km}, \quad \boxed{w > 0} \quad \square$$

Find every-thing

Ex. 3 Given $\triangle ABC$, where $A = 48^\circ$, $b = 17 \text{ cm}$ and $c = 25 \text{ cm}$, solve the triangle.



① side 'a'

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 17^2 + 25^2 - 2(17)(25) \cos 48$$

$$\boxed{a \approx 18.6 \text{ cm}}$$

18.6

② Find $\angle B$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{17} = \frac{\sin 48}{18.6}$$

$$\sin B = \frac{17 \sin 48}{18.6}$$

$$B = \sin^{-1} \left(\frac{17 \sin 48}{18.6} \right)$$

$$\begin{aligned} \angle C &= 180 - \\ &43 - 48 \\ &\approx 89^\circ \end{aligned}$$

$$B = 43^\circ$$

ANS

$$\frac{\sin C}{25} = \frac{\sin 48}{18.6}$$

$$\sin C = \frac{25 \sin 48}{18.6}$$

$$C = \sin^{-1} \left(\frac{25 \sin 48}{18.6} \right)$$

$$C \approx 87^\circ$$

Sine Law

vs.

Cosine Law

Use when you have an angle-side pair

Use when you have 2 sides and a CONTAINED angle or 3 sides (tomorrow's lesson)

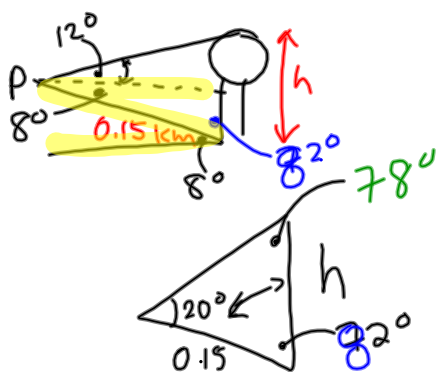
Homework

Page 409 # C1, C2,3,4b,7-9,11,15

Homework

Page 409 # C1, C2, 3, 4b, 7-9, 11, 15

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$$\frac{h}{\sin 20} = \frac{0.15}{\sin 108}$$

