7.2 Maximum Area for a Given Perimeter

Example 1: Bluebook page 33.

2.1.1: The Garden Fence

Problem

Your neighbour has asked for your advice about building his garden.

He wants to fence the largest rectangular garden with

20 metres of fencing.

Clarify the Problem

What are you being asked to determine?

Largest AREA

What information is useful?

Perimeter is 20 m

Explore

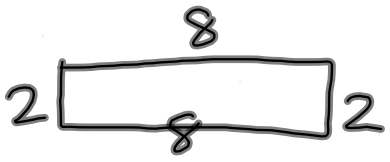
Use a geoboard or diagrams to show a model of one possible rectangular garden.

Hypothesize

What do you think the largest rectangular garden will look like? Sketch a picture of it with the dimensions. Calculate the area and perimeter.

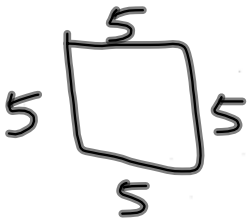
A = l x w

P = 20



20 - 4 = 16

16 ÷ 2



A = 25

33

2.1.1: The Garden Fence (continued)

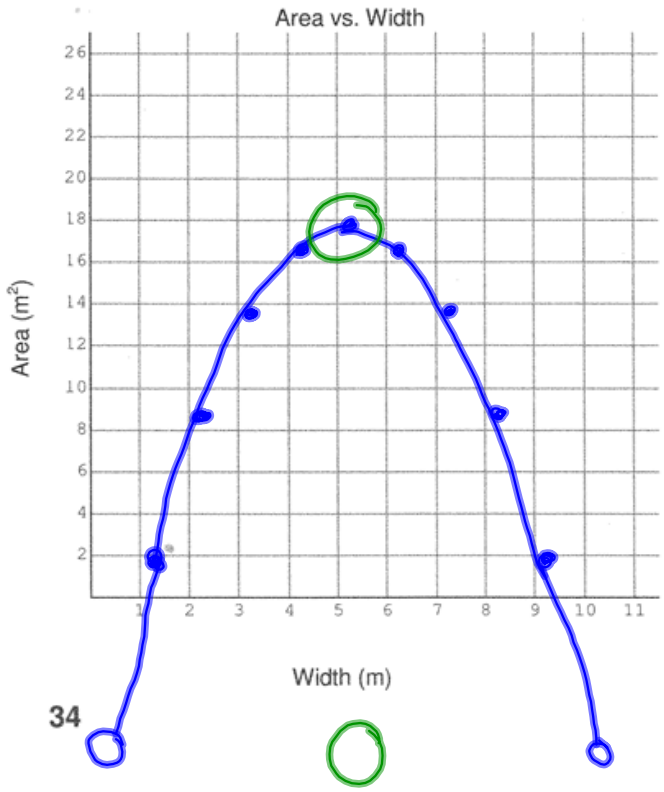
10
10

Model

Use the geoboard or diagrams to help you complete the table of values for the garden.

Perimeter (m)	Width (m)	Diagram	Length (m)	Area (m^2) $l \times w$
20	1	9	9	1x9 = 9
20	2	8	8	2x8 = 16
20	3			
20	4			
20	5	5	5	25
20	6			
20	7			
20	8			
20	9			
	10			

Construct a scatter plot of area vs. width.



Describe what happens to the area when the width of the garden increases.

As width ↑
Area increases, to a maximum then decreases

2.1.1: The Garden Fence (continued)

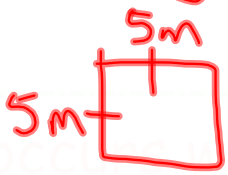
Manipulate
Look at the scatter plot.
Circle the region on the scatter plot where the area of the garden is the largest.
Construct two more sketches of garden areas with lengths and areas in this region.

Add these points to the scatter plot.

Conclude
What are the best dimensions for the garden? Justify your choice. Include a sketch and the area of the garden that you are recommending.

Best Dimensions: 5m by 5m

SQUARE



Maximum Area

The greatest area occurs when the rectangle is a square!

P = 20m

s = 20 ÷ 4

s = 5

P = 100 m

S = 100 ÷ 4

S = 25 m

7.2 Maximum Area for a Given Perimeter

Ex. 2: What is the maximum area of a rectangle with perimeter of 40 m?

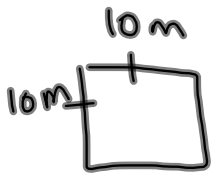
THINK

1. A square gives max area.
2. ∴ the 4 sides must be equal.
3. Divide the perimeter by 4 to get side length.
4. Use A = length x width.

$S = P \div 4$

$S = 40 \div 4$

$S = 10\text{m}$



$A = l \times w$

$= 10 \times 10$

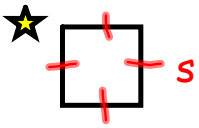
$= 100\text{m}^2$

Ex. 3 : Alex has 22 m of fencing. What dimensions will give the greatest area if the fencing is in 1 m secons and cannot be cut?

max

THINK

1. A square gives max area.



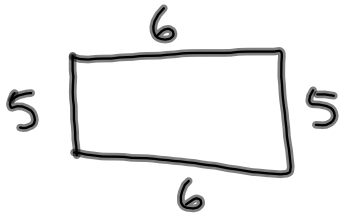
2. ∴ the 4 sides must be equal.

3. Divide the perimeter by 4 to get side length.

- $s = 22 \div 4$
- $s = 5.5$
- But we can't have decimal amounts!

When it is not possible to make a square, the maximum area occurs when the length and width are closest in value as possible.

- Choose length = 5 m and width = 6 m



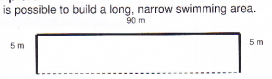
∴ Dimensions are 5m by 6m

Example 4: Bluebook page 39.

2.3.1: Down by the Bay

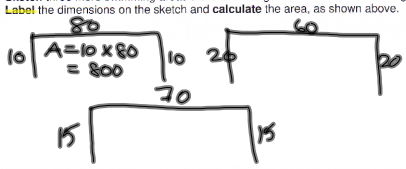
The city planners would like you to design a swimming area at a local beach. There is 100 m of rope available to enclose the swimming area. The shore will be one side of the swimming area; so only three sides of the rectangle will be roped off. It is your job to design the largest rectangular swimming area.

Explore
It is possible to build a long, narrow swimming area.



Area = length × width
Area = 90 x 5
Area = 450 m²

Sketch three more swimming areas that have a larger area than this swimming area.
Label the dimensions on the sketch and **calculate** the area, as shown above.



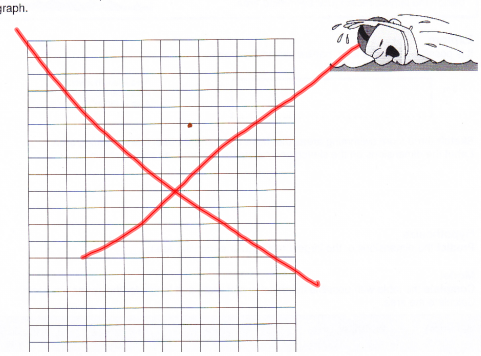
Model
Complete the table with possible combinations of width and length for the swimming pools.
Calculate the area.

Perimeter (m)	Width, w, (m)	Diagram	Length, l, (m)	Area, A, (m^2) l × w
100	0		100	0
100	5		90	450
100	10		80	800
100	15		70	1050
100	20		60	1200
100	25		50	1250
100	30		40	1200
100	35		30	1050
	40		20	800
	45		10	450
	50		0	0

33.3

Describe what happens to the area when the width of the swimming area increases.

Construct a scatter plot of area vs. width. Choose appropriate scales. **Label** and **title** your graph.



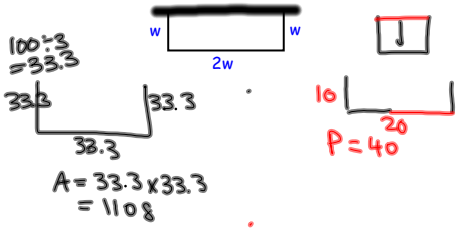
Manipulate
Circle the region on the scatter plot where the area of the swimming area is the largest.

Construct two more sketches of swimming areas with widths and areas in the circled region.
Add these points to the scatter plot.

Conclude
Write a report to the town advising them of the dimensions that would be best for the new swimming area. **Justify** your choice. **Include** a sketch and the area of the swimming area that you are recommending.

P= 100m
w=100÷4
w = 25
l = 25x2
= 50

★ When only 3 sides of a rectangle are available for perimeter, the optimal shape is a rectangle whose length is 2 times its width.



P = 80
P = 200 — 3 sides
What are the dimensions?

HOMEWORK

p. 55 #2, 3, 6

p. 67 #1, 3