7.3 Minimum Perimeter for a Given Area
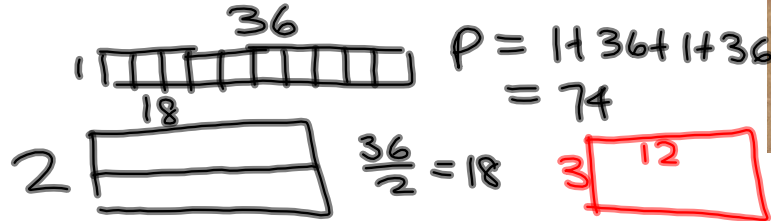
Invesgaon: page 61 in text.

Michael has 36 square stones to arrange as a rectangular pao. He will then buy edging to go around the pao.

- Sketch a few of the different paos that can be created.

$P = 14 \ 36 + 1 + 36$
$= 74$

$\frac{36}{2} = 18$

$36 \div 5 = 6.1$

- Which pao requires the least amount of edging?

Area	Length	Width Divide area by the length	Perimeter $P = 2\,l + 2\,w$
36	1	$36 \div 1 = 36$	$2(1) + 2(36) = 74$
36	2	18	$2(2) + 2(18) = 40$
36	3	12	30
36	4	9	26
36	5	7.2	Don't use 5 as we can't have part of a stone.
36	6	6	24	Minimum perimeter ⬅

$A = 36 \qquad \sqrt{36}$

➡ For rectangles with a <u>fixed</u> area, a square has the <u>minimum perimeter</u>.

- Suppose each stone has a side length of 30 cm.
  What is the least amount of edging needed for the pao?

30

30

Stone 30×6
$= 180$ cm

Patio

30×6
$= 180$ cm
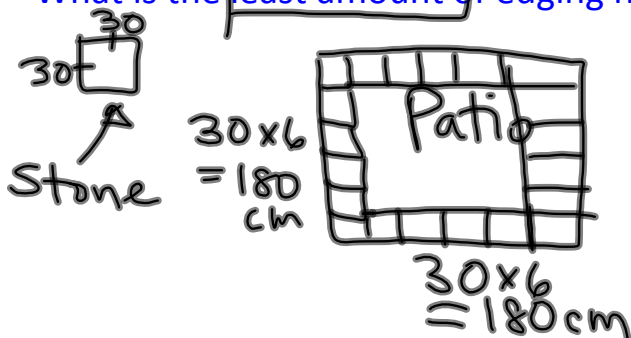
Edging
$=$ Perimeter
$= 180 + 180 + 180 + 180$
(or 4×180)

Ex. 1: For 75 pao stones, what are the dimensions that give the minimum
        perimeter?

THINK

Minimum perimeter = square.
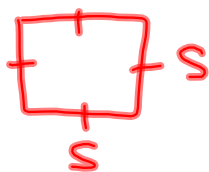∴ make a chart with factors of 75.
   (numbers that multiply to 75)

length	width	perimeter
1	75	2(1) + 2(75) = 152
3	25	56
5	15	2(5) + 2(15) = 40
15	5	repeat

➡ If it is not possible to form a square, (sides are restricted to whole numbers),
   then the minimum perimeter occurs when the rectangle is closest to a square.

Ex. 2: What is the minimum perimeter of a rectangle having an area of:

a) 121 cm²

∴ SQUARE

$A = S^2$
$121 = S^2$
$\sqrt{121} = S$
$11\,cm = S$

∴ $P = 11 + 11 + 11 + 11$
$= 4(11)$
$= 44$

b) 90 cm²

SQUARE

$S = \sqrt{90}$
$\doteq 9.5\ cm$

$P = 4(9.5)$
$\doteq 38\,cm$

Ex. 3: A pao is to be built on the side of a house using 24 congruent square
       stones. It will then be edged on 3 sides. Which arrangement requires
       the minimum edging?

THINK

length	width	perimeter
1	24	$2(1) + 24 = 26$
2	12	$2(2) + 12 = 16$
3	8	$2(3) + 8 = 14$
4	6	$2(4) + 6 = 14$
6	4	$2(6) + 4 = 16$
8	3	$2(8) + 3 = 19$
12	2	$2(12) + 2 = 26$
24	1	$2(24) + 1 = 49$

24 [1

12 [2

8 [3

3 [8

3 [P = 14] 8

4 [P = 14] 6
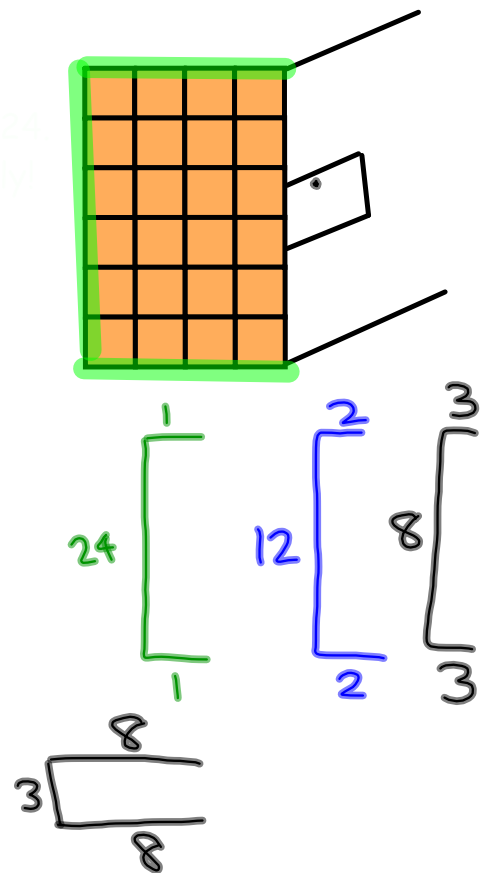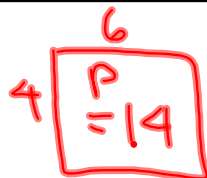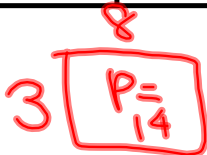
HOMEWORK
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